Lecture No-13: Topology of Metric Spaces

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Shri. V.G. Patil Saheb
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Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's

Arts,Commerce & Science College,Onde

Tal. Vikramgad,Dist. Palghar (MS)-401605

(Affiliated to the University of Mumbai)

NAAC Accrediated- Grade-C (CGPA-1.85)

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Topology of Metric Spaces

Unit I : Metric Spaces

Lecture - 13

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Theorem :- Let (X, d) be a metric space, the finite intersection of open sets is open

OR

If G_1 and G_2 are open subsets of metric space (X, d). Then $G_1 \cap G_2$ is open set.

Proof :- If $G_1 \cap G_2 = \phi$ then result is true. since ϕ is open set.

Let $G_1 \cap G_2 \neq \phi$.

Take $x \in G_1 \cap G_2$.

Take $x \in G_1 \cap G_2$. Then

$x \in G_1$ & $x \in G_2$.

Here G_1 is open set & $x \in G_1$

$\therefore \exists\ \varepsilon_1 > 0$ such that $B(x, \varepsilon_1) \subset G_1$.

Similarly, G_2 is open set & $x \in G_2$

$\therefore \exists\ \varepsilon_2 > 0$ such that $B(x, \varepsilon_2) \subset G_2$

Take $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$.

Then $B(x, \varepsilon) \subset G_1$ & $B(x, \varepsilon) \subset G_2$

$\therefore B(x, \varepsilon) \subset G_1 \cap G_2$

$\therefore G_1 \cap G_2$ is open set in X.

Remark :- Infinite intersection of open sets need not be open.

Justification :- Let $I_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$; for $n \in I$ the open interval in R^1.

Open intervals is open set in R^1.

$\therefore$ I_n is open set in R^1.

Now, $\bigcap_{n=1}^{\infty} I_n = \{0\}$

we know that one point set in R^1 is not open

$\therefore$ Infinite intersection of open sets need not be open.

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Example :- Let A & B be open subsets of $\mathbb{R}^1$. Then prove that $A \times B$ is open set in $\mathbb{R}^2$.

Solution :- A is open set in $\mathbb{R}^1$.
Then for $x \in A$ $\exists$ $\varepsilon_1 > 0$ such that

$$B(x, \varepsilon_1) \subset A$$

Similarly, B is open set in $\mathbb{R}^1$.
Then for $y \in B$ $\exists$ $\varepsilon_2 > 0$ such that

$$B(y, \varepsilon_2) \subset B.$$

Lecture 13: Metric Spaces

Take $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$

$\therefore B(x, \varepsilon) \subset A$

& $B(y, \varepsilon) \subset B$

$\therefore B(x, \varepsilon) \times B(y, \varepsilon) \subset A \times B$

$\therefore B((x, y), \varepsilon) \subset A \times B$

$\therefore A \times B$ is open set in $\mathbb{R}^2$.