Lecture No-1: System of Linear Equations and Matrices

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My Inspiration
Shri. V.G. Patil Saheb
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Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
Arts,Commerce & Science College,Onde
Tal. Vikramgad,Dist. Palghar (MS)-401605
(Affiliated to the University of Mumbai)
NAAC Accrediated- Grade-C (CGPA-1.85)
ISO-9001:2015 Certified
Year of Establishment: 2002

Linear Algebra-I

Unit I: System of Equations, Matrices

Lecture 1

Prof. Santosh Shivlal Dhamone

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Semester - III

1. Calculus - III — Course Code - USMT 301 — 100 Mks
2. Linear Algebra-I : Course Code - USMT 302 - 100 Mks
3. Ordinary Differential Equations : ODE
   Course Code: USMT 303 - 100 Mks

+ Practicals based on USMT 301, 302, 303

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Scheme of Examination

75 : 25 Pattern

The performance of the learners shall be evaluated into two parts:

1. Internal Assessment of 25 marks
2. Semester End Examination of 75 marks

Total : 100 Marks

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Mathematics Paper - II

Linear Algebra - I

USMT : 302

Unit-1 :- System of Equations and Matrices-

Unit-2 :- Vector Spaces over R

Unit-3 :- Determinants , Linear Equations.

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System of Homogenous and Non-homogenous Linear equations :-

A system of m linear equations in n unknowns (or variables) say $x_1, x_2, x_3, \cdots, x_n$ as follows:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3$$
$$\vdots \quad \cdots \quad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m$$

where a_{ij} & b_i are constants, may be real or complex.

$a_{ij} \rightarrow$ 'a' is the element in i^{th} row & j^{th} column

$i = 1, 2, \cdots, m$ & $j = 1, 2, \cdots, n$

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The above system of linear equations written in matrix form as follows.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

$m \times n$ (row, column) — Coefficient Matrix A

$n \times 1$ — Variable Unknown Matrix X

$m \times 1$ — Constant or Solution Matrix B

$$A_{m \times n} X_{n \times 1} = B_{m \times 1}$$

$$AX = B$$

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$$AX = B$$

Where

A = Coefficient Matrix $= \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$

X = Variable or Unknown Matrix

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

& B = Solution Matrix OR Constant Matrix $= \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$

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Homogenous System of linear Equations :-

A homogenous system of m linear equations in n unknown is as follows

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = 0$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = 0$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = 0$$

In Short

If $AX = B$ is a system of linear equations then

(1) if $B = 0$ the system of linear equations is known as homogenous.

and

(2) If $B \neq 0$ then the system of linear equations is known as non-homogenous.

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(2) If $B \neq 0$ then the system of linear equations is known as non-homogenous.

For example :-

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - 2x_2 - x_3 = 0$$
$$2x_1 + x_2 + 3x_3 = 0$$

is homogenous system of linear equations

And $x_1 + 2x_2 - x_3 = 3$
$3x_1 - x_2 + 2x_3 = 1$
$2x_1 - 2x_2 + 3x_3 = 2$

is non-homogenous system of linear equations.