Lecture No-2: System of Linear Equations and Matrices

Santosh Shivlal Dhamone

Assistant Professor in Mathematics
Art's Commerce and Science College,Onde
Tal:- Vikramgad, Dist:- Palghar

santosh2maths@gmail.com

August 14, 2021

Contents

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
Arts,Commerce & Science College,Onde
Tal. Vikramgad,Dist. Palghar (MS)-401605
(Affiliated to the University of Mumbai)
NAAC Accrediated- Grade-C (CGPA-1.85)
ISO-9001:2015 Certified
Year of Establishment: 2002

Linear Algebra- I
Unit-I System of Equations, Matrices
Lecture- 2

Santosh Shivlal Dhamone

Assistant Professor in Mathematics

Arts Commerce and Science College, Onde

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

System of Linear Equations :-

An system of m linear equations in n variables say $x_1, x_2, x_3, \cdots, x_n$ as follows :-

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m
\end{aligned}$$

where a_{ij} & b_i are constants may be real or complex.

a_{ij} = 'a' is an element in 'ith' row & 'jth' column.

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\
\vdots \quad \cdots \quad \cdots \quad \cdots & \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m
\end{aligned}
$$

where a_{ij} & b_i are constants may be real or complex.

a_{ij} = 'a' is an element in 'i th' row & 'j th' column.

The above system is said to be Non-homogenous system of linear equations.

If all $b_i = 0$ i.e. $b_1 = b_2 = b_3 = \cdots = b_m = 0$ then given system is said to be Homogenous system of linear equations.

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

The above system of equations written in Matrix form as follows.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1}$$

Coefficient Matrix

Unknown or Variable Matrix

Constant or solution Matrix

Simply it is written as

$$AX = B$$

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Examples & Solution of Non-Homogenous System of Equations :-

(1) Solve following system of non-homogenous equations :-

$$x_1 + x_2 + x_3 = 2$$
$$6x_1 - 4x_2 + 5x_3 = 31$$
$$5x_1 + 2x_2 + 2x_3 = 13$$

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Solution:- We have

$$x_1 + x_2 + x_3 = 2 \quad \text{---} \quad (1)$$
$$6x_1 - 4x_2 + 5x_3 = 31 \quad \text{---} \quad (2)$$
$$5x_1 + 2x_2 + 2x_3 = 13 \quad \text{---} \quad (3)$$

First we eliminate x_1 from eqn (2) & (3) by using eqn (1)

To eliminate x_1 from eqn (2) we multiply eqn (1) by -6, we get

$$-6x_1 - 6x_2 - 6x_3 = -12 \quad \text{---} \quad (4)$$

Adding eqn (2) & (4), we get

$$\begin{array}{r} 6x_1 - 4x_2 + 5x_3 = 31 \\ +\ -6x_1 - 6x_2 - 6x_3 = -12 \\ \hline -10x_2 - x_3 = 19 \end{array}$$

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

$\therefore -10x_2 - x_3 = 19$ ——— (5)

To elimate x_1 from eqn (3) we multiply eqn (1) by -5, we get

$-5x_1 - 5x_2 - 5x_3 = -10$ ——— (6)

Adding eqn (3) & (6), we get

$$\begin{array}{r} 5x_1 + 2x_2 + 2x_3 = 13 \\ + \quad -5x_1 - 5x_2 - 5x_3 = -10 \\ \hline -3x_2 - 3x_3 = 3 \end{array}$$

$\therefore -x_2 - x_3 = 1$ ——— (7)

Lecture 2: System of Linear Equations and Matrices

Now system of equations are as below:-

$$x_1 + x_2 + x_3 = 2 \quad \text{—— (1)}$$

$$-10x_2 - x_3 = 19 \quad \text{—— (5)}$$

$$-x_2 - x_3 = 1 \quad \text{—— (7)} \times -10$$

Now, We elimate x_2 from eqn (7) by using eqn (5),

Multiply eqn (7) by -10, we get

$$10x_2 + 10x_3 - 10 \quad \text{—— (8)}$$

Adding eqn (5) & (8), we get

$$\begin{array}{r} -10x_2 - x_3 = 19 \\ +10x_2 + 10x_3 = -10 \\ \hline 9x_3 = 9 \end{array}$$

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Hence we get

$$9x_3 = 9$$

$$\Rightarrow x_3 = \frac{9}{9}$$

$$\Rightarrow \boxed{x_3 = 1}$$

By Back substitution, we get value of x_2 & x_4.

Put $x_3 = 1$ in eqn (5)

$$-10x_2 - x_3 = 19$$

$$\Rightarrow -10x_2 - 1 = 19$$

$$\Rightarrow -10x_2 = 19 + 1 \rightarrow -10x_2 = 20$$

$$\therefore x_2 = \frac{20}{-10}$$

$$\therefore \boxed{x_2 = -2}$$

Lecture 2: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Put $x_2 = -2$ & $x_3 = 1$ in eqn (1)

$x_1 + x_2 + x_3 = 2$

$\therefore x_1 - 2 + 1 = 2$

$x_1 - 1 = 2$

$\boxed{x_1 = 3}$

Hence, required solution of given system of eqns is

$\boxed{x_1 = 3,\ x_2 = -2 \text{ & } x_3 = 1}$

Check :- Consider L.H.S of eqn (1)

$L.H.S = x_1 + x_2 + x_3$

$= 3 - 2 + 1$

$= 2$

$= R.H.S$