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Lecture No-3: System of Linear Equations and Matrices

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Contents

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Arts,Commerce & Science College,Onde

Tal. Vikramgad,Dist. Palghar (MS)-401605

(Affiliated to the University of Mumbai)

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Linear Algebra- I

Unit I: System of Linear Equations, Matrices

Lecture 3

System of Non-homogenous Linear Equations

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Solution of Simultaneous linear equations By elementary row transformation :-

Let us consider a system of m linear equations in n unknowns say $x_1, x_2, \ldots, x_n$ as below :

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\cdots\cdots\cdots\cdots\cdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

where a_{ij} & b_i are constants, may be real or complex.

The above system of equations can be written in matrix form as:

$$AX = B$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{\text{Coefficient Matrix}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Unknown of Variables Matrix

Solution Matrix

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For example :-

$$4x_1 + 2x_2 = 5$$
$$3x_1 + 1x_2 = 7$$

The above system of equation written in matrix form $AX = B$ as follows:

$$\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$AX = B$$

Augumented Matrix :-

The matrix of the form $[A : B]$ or $[A|B]$ is known as Augumented matrix.

In above example :-

$$[A|B] = \left[\begin{array}{cc|c} 4 & 2 & 5 \\ 3 & 1 & 7 \end{array}\right] = \begin{bmatrix} 4 & 2 & : & 5 \\ 3 & 1 & : & 7 \end{bmatrix}$$

Remark :-

$\rho(A|B)$ — length of Augumeted Matrix.

$\rho(A)$ — length of Matrix A.

* **Consistancy & Solution of Non-homogeneous system of equations :-**

① $\rho(A|B) = \rho(A) =$ number of unknown (n)

Then equations are consistant & has unique solution.

② $\rho(A|B) = \rho(A) <$ number of unknown (n)

Then equations are consistant & has infinite solution.

③ $\rho(A|B) \neq \rho(A)$

Then equations are inconsistant & has no solution.

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* Consistency of Non-homogenous linear equations :-

By the solution of the system of equations we mean that to find a set of values of the unknowns $x_1, x_2, \cdots, x_n$ which satisfy all the given m equations.

But here we clear that linear equations do not always have a solution i.e. it is not always possible to find the values of the unknown.

Examples :-

1. Consider the set of equations

$$2x + 3y = 3$$
$$x - y = 1$$

Above system is system of 2 equations in 2 variables x & y.

Soln :- The given system of equations in written in matrix form $AX=B$ as

$$\begin{array}{c} \text{Row 1} = R_1 \\ \text{Row 2} = R_2 \end{array} \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$\quad x \qquad y$$

Consider augmented matrix $[A|B]$

$$\therefore [A|B] = \begin{array}{c} R_1 \\ R_2 \end{array} \left[\begin{array}{cc|c} 2 & 3 & 3 \\ 1 & -1 & 1 \end{array}\right]$$

a_{21}

$R_2 \downarrow -2R_2 + R_1$

$$\begin{array}{rrrr} -2R_2 = & -2 & 2 & -2 \\ R_1 + & 2 & 3 & 3 \\ \hline & 0 & 5 & 1 \end{array}$$

Examples:-

① Consider the set of equations

$$2x + 3y = 3$$
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Above system is system of 2 equations in 2 variables x & y.

Soln:- The given system of equations is written in matrix form $AX=B$ as

$$\begin{array}{r} \text{Row 1} = R_1 \\ \text{Row 2} = R_2 \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \\ \begin{matrix} x & \quad y \end{matrix} \end{array} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Consider augmented matrix $[A \mid B]$

$$\therefore [A \mid B] = \begin{array}{c} R_1 \\ R_2 \end{array} \left[\begin{array}{cc|c} 2 & 3 & 3 \\ 1 & -1 & 1 \end{array}\right]$$

a_{21}

$R_2 \to -2R_2 + R_1$

$$\begin{array}{rccc} -2R_2 = & -2 & 2 & -2 \\ R_1 + & 2 & 3 & 3 \\ \hline & 0 & 5 & 1 \end{array}$$

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$$\left[\begin{array}{cc|c} 2 & 3 & 3 \\ 0 & 5 & 1. \end{array}\right]$$

$\rho(A|B) = 2$ & $\rho(A) = 2$

n = number of unknown $= 2$

Here $\rho(A|B) = \rho(A) = n = 2$

Hence system is consistant & has unique solution.

$$\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$2x + 3y = 3$

$5y = 1 \Rightarrow y = \frac{1}{5}$

Using $y = \frac{1}{5}$ in eqn $2x + 3y = 3$, we get

$2x + 3 \times \frac{1}{5} = 3$

$\therefore 2x = 3 - \frac{3}{5} = \frac{15-3}{5} = \frac{12}{5}$

$\Rightarrow x = \frac{6}{5}$

$\therefore (x, y) = \left(\frac{6}{5}, \frac{1}{5}\right)$ is a solution of given system of eqn.

Check:- Consider L.H.S of eqn ①, L.H.S $= 2x + 3y$

$= 2 \times \frac{6}{5} + 3 \times \frac{6}{5}\ \frac{1}{5}$

$= \frac{12}{5} + \frac{8}{5}\ \frac{3}{5}$

$= \frac{30}{5}\ \ 15/5 = 3$

Consider L.H.S of eqn ①,

$$L.H.S = 2x + 3y = 2 \times \frac{6}{5} + 3 \times \frac{1}{5}$$
$$= \frac{12}{5} + \frac{3}{5}$$
$$= 15/5$$
$$= 3$$

L.H.S = R.H.S

IIIly, L.H.S of eqn ②

$$L.H.S = x - y = \frac{6}{5} - \frac{1}{5} = \frac{6-1}{5} = 5/5 = 1$$

$\therefore$ L.H.S = R.H.S

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② Consider the set of equations

$$2x + 3y = 7$$
$$8x + 12y = 28$$

Solution:- Given system of equations written in matrix form as

$$AX = B$$

$$\begin{array}{c} R_1 \\ R_2 \end{array}\begin{bmatrix} 2 & 3 \\ 8 & 12 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 28 \end{bmatrix}$$

a_{21}

Consider

$$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 7 \\ 8 & 12 & 28 \end{array}\right] \xrightarrow[R_2]{R_2 - 4R_1} \left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 0 & 0 \end{array}\right]$$

$\rho(A|B) = 1, \quad \rho(A) = 1, \quad n = 2$

Here $\rho(A|B) = \rho(A) = 1 < n = 2$

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$\rho(A|B) = 1, \quad \rho(A) = 1, \quad n = 2$

Here $\rho(A|B) = \rho(A) = 1 < n = 2$

$\therefore$ Given system of equations is consistent but has infinite solution

$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$

$2x + 3y = 7$

$\Rightarrow 2x = 7 - 3y$

$\Rightarrow x = \frac{7-3y}{2}$

Suppose $y = 1 \Rightarrow x = \frac{7-3}{2} = 2$

$y = -10 \Rightarrow x = \frac{7+30}{2} = 37/2$

& so on.

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③ Consider the set of equations

$2x+3y=7$

$4x+6y=20 \Rightarrow$ dividing by 2 $\quad 2x+3y=10$

$(x,y) \Rightarrow \quad 2x+3y=7$
$\qquad\qquad\qquad\quad =10$

No solution.

Soln:- i) We write above system of equations in matrix form as

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 20 \end{bmatrix}$$

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Consider $[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 7 \\ 4 & 6 & 20 \end{array}\right]$

$\downarrow R_2 - 2R_1$

$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 0 & 6 \end{array}\right] = 2$

$\therefore \rho(A|B) = 2$, $\rho(A) = 1$

Here $\rho(A|B) \neq \rho(A)$.

Hence given system is inconsistant & has no solution.

Revision :-

1. $\rho(A|B) = \rho(A) = n$, Consistant Unique solution
2. $\rho(A|B) = \rho(A) < n$ Consistant Infinite Solution
3. $\rho(A|B) \neq \rho(A)$ Inconsistant No Solution

Thank You! ☺