My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Lecture No-4: System of Linear Equations and Matrices

Santosh Shivlal Dhamone

Assistant Professor in Mathematics
Art's Commerce and Science College,Onde
Tal:- Vikramgad, Dist:- Palghar

santosh2maths@gmail.com

August 24, 2021

Contents

Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's

Arts,Commerce & Science College,Onde

Tal. Vikramgad,Dist. Palghar (MS)-401605

(Affiliated to the University of Mumbai)

NAAC Accrediated- Grade-C (CGPA-1.85)

ISO-9001:2015 Certified

Year of Establishment: 2002

Linear Algebra- I

Unit I: System of Equations, Matrices

Lecture 4

Methods of Solving Non-Homogenous System: Gaussian Elimination Method: AX=B

Santosh SWhivlal Dhamone

Assistant Professor in Mathematics

Arts Commerce and Science College, Onde

Methods of Solving Non-Homogenous System :-
Gauss Elimination Method : $AX = B$, where $B \neq 0$

(1) If $\rho(A|B) = \rho(A)$ = number of unknown (n) then;
System is consistant with unique solution.

(2) If $\rho(A|B) = \rho(A) <$ number of unknown (n), then;
system is consistant with infinite solutions
& in that case $(n-r)$ variables are assigned arbitrary values; where n = no. of unknown & r = rank

(3) If $\rho(A|B) \neq \rho(A)$ then system is inconsistant & has no solution.

Note :-

(1) Try to convert augumeted matrix into an row echelon form using elementry row operations only.

(2) Then find values of unknown by using backward substitution method.

Examples :-

(1) Test for consistancy and solve following system of equation

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

Lecture 4: System of Linear Equations and Matrices

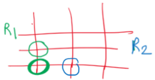
My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Solution :- We write given system of equations in Matrix form

$$AX = B$$

$$\therefore \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$



Consider

$$[A|B] = \begin{matrix} 3 \times R_1 \\ 2 \times R_2 \\ R_3 \end{matrix} \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$$\Big\downarrow \; -2R_2 + 3R_1, \quad R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & -11 & 27 & 11 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

-26
$+15$

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & -11 & 27 & 11 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$\downarrow R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & -11 & 27 & 11 \\ 0 & 0 & 0 & 49 \end{array}\right]$$

$\rho(A) = 2$

Here $\rho(A|B) = 3$, $\rho(A) = 2$

$\therefore \rho(A|B) \neq \rho(A)$

In this case system has no solution.

Hence given system has no solution.

My Inspiration Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

(2) Test for consistancy & solve the following system of equations:-

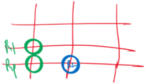
$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

Solution:- We write given system of equations in matrix form

$$AX = B$$

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

a_{21}, a_{31}, a_{32}



We consider,

$$[A|B] = \begin{bmatrix} 5 & 3 & 7 & | & 4 \\ 3 & 26 & 2 & | & 9 \\ 7 & 2 & 10 & | & 5 \end{bmatrix}$$

($-7 \times$, $-3 \times$, $5 \times$, $5 \times$)

$\downarrow$ $5R_2 - 3R_1$, $5R_3 - 7R_1$

−15	−9	−21	−12
15	130	10	45
0	121	−11	33

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$$

$$\Big\downarrow \; 11R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A|B) = 2$

Here, $\rho(A) = 2$, $\rho(A|B) = 2$ & n = no. of unknown $= 3$

$\therefore \rho(A) = \rho(A|B) < n = 3$

Lecture 4: System of Linear Equations and Matrices

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

In this case system is consistent with infinite sol^n
& we take $(n-r) = 3-2 = 1$ variable is arbitrary.

$$\begin{bmatrix} 5 & 3 & 7 \\ 0 & 124 & -11 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 33 \\ 0 \end{bmatrix}$$

we get

$$5x + 3y + 7z = 4 \quad \text{———} \quad (*)$$

$$124y - 11z = 33$$

$$\Rightarrow 124y = 11z + 33$$

$$\Rightarrow y = \frac{11(z+3)}{124}$$

$$\Rightarrow y = \frac{z+3}{11}$$

Put $z = t$; any arbitrary value

$$\therefore \boxed{y = \frac{t+3}{11}}$$

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Put $y = \frac{t+3}{11}$ & $z = t$ in eqn (*)

$$5x + 3\left(\frac{t+3}{11}\right) + 7t = 4$$

$$\therefore 5x + \frac{3t+9}{11} = 4 - 7t$$

$$5x + \frac{3t+9}{11} = 4 - 7t$$

$$55x + 3t + 9 = 44 - 77t$$

$$55x = 44 - 77t - 3t - 9$$

$$\boxed{x = \frac{33 - 80t}{55}}$$

$\therefore x = \frac{33-80t}{55}$, $y = \frac{t+3}{11}$ & $z = t$ is set of solution

Thank You