Lecture No-5: System of Linear Equations and Matrices

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Contents

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Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
Arts,Commerce & Science College,Onde
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Linear Algebra- I
Unit-I: System of Equations, Matrices

Lecture 5

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My Inspiration
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Revision :-

Methods of solving non-homogenous system $AX=B$; where B

1. If $\rho(A)=\rho(A|B)$ = number of unknown (n); then system is consistant with unique solution.
2. If $\rho(A)=\rho(A|B)<$ number of unknown (n); then system is consistant with infinite solution.
3. If $\rho(A)\neq\rho(A|B)$ then system is inconsistant & has solution.

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Examples :-

(1) Show that the system of equations

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$
$$2x - 3y - z = 5$$

are consistant & hence solve it.

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Solution :- The given system of equation, write in matrix form $AX = B$ as follows:

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix}$$

Consider,

$$[A|B] = \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right] \xrightarrow[3R_4 - 2R_1]{3R_2 - R_1} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \left[\begin{array}{ccc} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 10 & 3 \\ 0 & -15 & -7 \end{array}\right.$$

$$\Big\downarrow \begin{array}{l} 3R_3 - \\ 3R_4 + \end{array}$$

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Consider,

$$[A|B] = \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right] \xrightarrow[3R_4 - 2R_1]{3R_2 - R_1} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 10 & 3 & -2 \\ 0 & -15 & -7 & 13 \end{array}\right]$$

$$\xrightarrow[3R_4 + 15R_2]{3R_3 - 10R_2} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & -51 & 204 \end{array}\right]$$

$$\xrightarrow{29R_4 + 51R_3} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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Here $\rho(A|B) = 3$, $\rho(A) = 3$, no. of unknown $= 3$

$\therefore \rho(A) = \rho(A|B) = n = 3$

Hence given system is consistent & has unique solution.

$$\therefore \begin{bmatrix} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & 29 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \\ -116 \\ 0 \end{bmatrix}$$

By back substitution method,

$29z = -116 \Rightarrow z = \frac{-116}{29} = -4$

$3y - 2z = 11$

$\therefore 3y - 2\times -4 = 11 \Rightarrow 3y = 11 \oplus 8 =$

$3y + 8 = 11 \Rightarrow 3y = 11 - 8 = 3 \Rightarrow y = 3/3 = 1$

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$3y - 2z = 11$

$\therefore\ 3y - 2\times -4 = 11 \Rightarrow 3y = 11 + 8 =$

$3y + 8 = 11 \Rightarrow 3y = 11 - 8 = 3 \Rightarrow y = 3/3 = 1$

$3x + 3y + 2z = 1 \Rightarrow 3x + 3\times1 + 2\times -4 = 1$

$\Rightarrow 3x + 3 - 8 = 1$

$\therefore\ 3x - 5 = 1$

$3x = 6$

$x = 6/3$

$\therefore\ x = 2$

Required solution of given system of equations is

$x = 2,\ y = 1\ \&\ z = -4.$

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(2) Find for what values of λ & μ, the equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

Have (i) no solution

(ii) A unique solution

(iii) infinite number of solution.

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Solution:- We write given system of equations in matrix form

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

Consider, augmented matrix $[A|B]$.

$$[A|B] = \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$$\Big\downarrow \; R_2 - R_1, \; R_3 - R_1$$

$$\begin{array}{c} \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

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$$\begin{array}{c c} & \\ R_2 & \\ R_3 & \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$$\Big\downarrow R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$$\begin{array}{cc} 0 & 0 \end{array}$$

(i) No solution.

$$\rho(A) \neq \rho(A|B)$$

$$2 \neq 3$$

$\therefore$ $\lambda - 3 = 0 \Rightarrow \lambda = 3$

& $\mu - 10 \neq 0 \Rightarrow \mu \neq 10$

$\therefore$ $\mu \neq 10$ & $\lambda = 3$

(ii) A unique sol^n.

If system of equation has unique solution

$\therefore \quad \rho(A) = \rho(A|B) =$ no. of unknown.

$\therefore$ In this case $\lambda - 3 \neq 0$ & $\mu - 10 \neq 0$

$$\therefore \quad \lambda \neq 3 \quad \& \quad \mu \neq 10.$$

(iii) infinite number of solution.

We know that system of equation has infinite solution

when $\rho(A) = \rho(A|B) <$ number of unknown

$$\rho(A) = \rho(A|B) = 2 < 3$$

$$\therefore \quad \lambda - 3 = 0 \quad \& \quad \mu - 10 = 0$$

$$\therefore \quad \lambda = 3 \quad \& \quad \mu = 10.$$