Lecture No-6: System of Linear Equations and Matrices

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Contents

Lecture 6: System of Linear Equations and Matrices

Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
Arts,Commerce & Science College,Onde
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Linear Algebra-I

Unit I: System of Linear Equations, Matrices

Lecture 6

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Non-Homogenous System of linear equations :-

Let $AX = B$ be a system of linear equations in matrix form. If $B \neq 0$ then given system of equations is non-homogenous.

(1) If $\rho(A|B) = \rho(A) = n$;

where $\rho(A|B)$ — length of augmented matrix $[A|B]$

$\rho(A)$ — length of matrix A

n — number of unknown

then given system of non-homogenous linear equations is consistant & has unique solution.

(2) If $\rho(A|B) = \rho(A) < n$ then system is consistant but it has infinitely many solution.

(3) If $\rho(A|B) \neq \rho(A)$ then system of non-homogenous linear equations is inconsistant & has no solution.

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My Inspiration Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

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Examples :-

(5) For what values of λ, the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

have a solution & solve completely in each case.

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Sol^n :- Given system of equations is

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

We write given system of equations in matrix form

$$AX=B$$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

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Consider augmented matrix $[A|B]$

$$\therefore \quad [A \mid B] = \begin{matrix} R_1 \to \\ R_2 \to \\ R_3 \to \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

Applying Row operations on $[A|B]$

$$\downarrow \; R_2 - R_1, \; R_3 - R_1$$

$$\begin{matrix} \\ R_2 \\ R_3 \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 3 & 9 & \lambda^2 - 1 \end{array}\right]$$

$$\downarrow \; R_3 - 3R_2$$

$R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & (\lambda-1)(\lambda-2) \end{array}\right]$$

$(\lambda^2-1) - 3(\lambda-1)$

$\Rightarrow (\lambda-1)(\lambda+1) - 3(\lambda-1)$

$\Rightarrow (\lambda-1)[\lambda+1-3]$

$\Rightarrow (\lambda-1)(\lambda-2)$

Case (1) :- If $\rho(A) = \rho(A|B) = n$ = no. of unknown

then it has unique solution.

Here $\rho(A) = 2$ & $n = 3$

Hence $\rho(A) = 2 < n = 3$

So there does not exist unique solution

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Case (2) :- If $\rho(A|B) = \rho(A) < n$ then it has infinitely many solutions.

Here $\rho(A) = 2$, $n = 3$ & $\rho(A|B) = 3$

To make $\rho(A|B) = 2$, we take or

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\Rightarrow \lambda - 1 = 0 \quad \text{or} \quad \lambda - 2 = 0$$

$$= \quad = \quad \text{or} \quad = 2$$

$\therefore$ Given system has infinitely many solutions when $\lambda = 1$, $\lambda = 2$

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Case (3):- No Solution

If $\rho(A) \neq \rho(A|B)$ then system of equation has no solution.

Here $\rho(A) = 2$ & $\rho(A|B) = 3$

when $(\lambda-1)(\lambda-2) \neq 0$

$\Rightarrow \lambda \neq 1$ & $\lambda \neq 2$

$\therefore$ When $\lambda \neq 1, 2$ then given system has no solution

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(6) Investigate the values of λ & μ so that the equations:

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have (i) no solution

(ii) a unique solution

(iii) an infinite number of solution.

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Solution :- Write given system of equations in matrix form $AX = B$ as follows

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

Consider Augmented matrix $[A|B]$ & apply row operations on it.

To make first entry in row second & third (i.e. $a_{21}=0$ & $a_{31}=0$) is equal to zero using first

Consider

$$[A|B] = \begin{matrix} R_1 \to \\ R_2 \to \\ R_3 \to \end{matrix} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ \textcircled{7} & 3 & -2 & 8 \\ \textcircled{2} & 3 & \lambda & \mu \end{array}\right]$$

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Consider

$$[A \mid B] = \begin{matrix} R_1 \to \\ R_2 \to \\ R_3 \to \end{matrix} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$\Big\downarrow \begin{matrix} 2R_2 - 7R_1 \\ R_3 - R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

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(i) No solution :-

If $\rho(A|B) \neq \rho(A)$ then system of equation has no solut

Here $\rho(A|B) = 3$ & $\rho(A) = 3$

$\therefore$ If $\mu - 9 = 0 \Rightarrow \mu = 9$ then $\rho(A|B) = 2$

& $\rho(A) = 2$ when $\lambda - 5 = 0 \Rightarrow \lambda = 5$.

Hence if $\lambda = 5$ then given system has no solutio

because $\rho(A) = 2 \neq \rho(A|B) = 3$

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(iii) a unique solution :-

For unique solution

$$\rho(A|B) = \rho(A) = n$$

Here $n = 3$

$$\therefore \rho(A|B) = \rho(A)$$

$$\Rightarrow \lambda - 5 \neq 0 \ \& \ \mu - 9 \neq 0$$

$$\Rightarrow \lambda \neq 5 \ \& \ \mu \neq 9$$

(iii) an infinite solution

For an infinite solution

$$\rho(A|B) = \rho(A) < n$$

Here $n = 3$

$\therefore \quad \rho(A|B) = \rho(A) = 2$

$\Rightarrow \quad \lambda - 5 = 0 \quad \& \quad \mu - 9 = 0$

$\Rightarrow \quad \lambda = 5 \quad \& \quad \mu = 9$

For $\lambda = 5 \qquad \rho(A) = 2$

$\& \quad \mu = 9 \qquad \rho(A|B) = 2.$

$\therefore$ For an infinite solution $\mu = 9$ & $\lambda = 5$.