Lecture No-7: System of Linear Equations and Matrices

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August 26, 2021

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Shri. V.G. Patil Saheb
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Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
Arts,Commerce & Science College,Onde
Tal. Vikramgad,Dist. Palghar (MS)-401605
(Affiliated to the University of Mumbai)
NAAC Accrediated- Grade-C (CGPA-1.85)
ISO-9001:2015 Certified
Year of Establishment: 2002

Linear Algebra - I

Unit I : System of Equations, Matrices
Lecture 7

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Lecture 6: System of Linear Equations and Matrices

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System of linear Equations

How to write system of equations in Matrix form

$$AX = B$$

Non-Homogenous Systems of linear Equations

Solution of Non-Homogenous System of linear equations

Problems on Non-Homogenous System of linear equations

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Homogeneous System of Linear Equations & its Solution :-

The system $AX = B$ of m linear equations in n unknowns is known as the system of homogeneous linear equations if $B = 0$

Thus the above equation reduces to

$$AX = 0$$

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For such a system, matrix A & the augmented matrix $[A|B]$ or $[A : B]$ are the same.

So that the rank of coefficient matrix A i.e. $S[A]$ and is augmented matrix i.e. $S[A|B]$ are equal

i.e $\boxed{S[A|B] = S[A]}$

Hence a system of homogeneous linear equations— is ways consistent.

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Solution of Homogeneous System of Linear Equations :-

(1) Trivial Solution :-

If $\rho[A|B] = \rho[A] = n$; where n is number of unknowns, $\rho[A|B]$ = Rank of Augmented Matrix or length of Augmented Matrix

& $\rho[A]$ = Rank of Matrix A or length of Matrix A.

$$\rho[A|B] = \rho[A] = n$$

Then system is consistent & it has unique solution.

The above solution is known as Trivial Solution.

$X = 0$ i.e. $x_1 = 0, x_2 = 0, \ldots, x_n = 0$

is always a solution of the given system of equations.

This solution is called Trivial solution or zero solution.

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For example :- Consider a system of equations

$$2x - y = 0$$
$$x + y = 0$$

This is homogenous system of linear equations.

We write given system of eqns in matrix form $AX = B$ as follows

$$\therefore \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let us consider,

$$[A|B] = \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right] \begin{matrix} R_1 \\ R_2 \end{matrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\downarrow 2R_2 - R_1$$

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Let us consider,

$$[A|B] = \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right] = \begin{matrix} R_1 \\ R_2 \end{matrix}\left[\begin{array}{cc} 2 & -1 \\ 1 & 1 \end{array}\right]$$

$$\downarrow 2R_2 - R_1$$

$$\left[\begin{array}{cc} 2 & -1 \\ 0 & 3 \end{array}\right]$$

$S[A] = 2$, $S[A|B] = 2$, $n = 2$

$\therefore S[A|B] = S[A] = n$

For solution

$$\left[\begin{array}{cc} 2 & -1 \\ 0 & 3 \end{array}\right]\left[\begin{array}{c} x \\ y \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \end{array}\right]$$

$\therefore 2x - y = 0$

& $3y = 0 \Rightarrow y = 0$

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& $3y = 0 \Rightarrow y = 0$

$\therefore 2x - 0 = 0 \Rightarrow x = 0$

Hence $x = 0$, $y = 0$ is trivial solution of given system of equations.

For MCQ or Objective Type Question:-

It is sufficient to find determinant of matrix A i.e. $|A|$ & if $|A| \neq 0$ then system has trivial solution.

For above example:-

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = (2 \times 1) - (-1 \times 1) = 2 + 1 = 3 \neq 0$$

since $|A| \neq 0$ hence given system of equation has trivial solution.

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(2) Non-Trivial Solution :-

If $S[A|B] = S[A] < n$ then system is consistant & it has infinetly many solutions.

This type of solution is known as Non-Trivial solution.

The system will have Non-Trivial solution if and only if $S[A|B] = S[A] < n$

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For Example :-

Consider a homogenous system of linear equations

$$2x - y = 0$$
$$4x - 2y = 0$$

We write above system of equations in matrix form $AX = B$ or

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Consider,

$$[A|B] = [A] = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

Here $\rho[A] = 1$ & $\rho[A|B] = 1$

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Here $\rho[A] = 1$ & $\rho[A|B] = 1$

$n = 2$

$\therefore \rho[A|B] = \rho[A] = 1 < n = 2.$

Hence given system of equations has non-trivial solution.

Consider, $\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x - y = 0$

$\Rightarrow 2x = y$

$\Rightarrow x = y/2$

Put $y = k$

$\therefore x = \frac{k}{2}$; when k is any integer.

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Consider, $\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x - y = 0$

$\Rightarrow 2x = y$

$\Rightarrow x = y/2$

Put $y = k$

$\therefore x = \frac{k}{2}$; when k is any integer.

For MCQ it is sufficient to check $|A| = 0$.

$\therefore |A| = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = -4 + 4 = 0$

$|A| = 0$

Hence given system has non-trivial solution.