Lecture No-8: System of Linear Equations and Matrices

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My Inspiration Shri. V.G. Patil Saheb
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Linear Algebra - I

Unit- I : System of Linear Equations, Matrices

Lecture 8

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Methods of Solving Homogeneous System of Linear Equations :-

A system of linear equations $AX = B$ is homogeneous if $B = 0$.

(1) If $\rho[A|B] = \rho[A] = n$, then system is consistant with unique solution or Trivial solution

(2) If $\rho[A|B] = \rho[A] < n$; then system is consistant with infinitely many solutions i.e. non-trivial solution. Also exist & in that case $(n-r)$ variables are assigned arbitary values.

Where n = no. of unknowns

& r = rank of matrix $A = \rho[A]$

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Note :-

(1) Homogeneous system is always consistant because in that case rank of coefficient matrix is always equal to rank of augmented matrix i.e. $\rho[A] = \rho[A|B]$.

(2) Trivial solution means $(x, y, z) = (0, 0, 0) \approx$ zero solution.

(3) Non-Trivial solution means non-zero solution.

(4) When the number of equations is less than the number of unknowns, the Homogeneous system will always have an infinite number of solution.

(4) When the number of equations is less than the number of unknowns, the Homogeneous system will always have an infinite number of solution.

e.g. $x + y + z = 0$

$x - 2y - z = 0$

In given system there are two equations & three unknowns.

Hence it has infinite number of solution.

Problems :-

(1) Solve the system of equations using matrix method

$$x + 3y - z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

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Solution:- We write given system of equations in matrix form $AX = B$

$$\therefore \underbrace{\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}}_{\text{Coefficient Matrix}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{Variables or Unknown Matrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Consider,

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$$= \begin{array}{c} R_1 \to \\ R_2 \to \\ R_3 \to \end{array} \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

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Consider,

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$$= \begin{array}{c} R_1 \to \\ R_2 \to \\ R_3 \to \end{array} \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

$$\Big\downarrow \begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & -7 & 6 \\ 0 & -14 & 15 \end{bmatrix}$$

$$\Big\downarrow R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & -7 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

Here $\rho[A|B] = \rho[A] = 3 = n$

$\therefore$ Given system is consistant & it has trivial solution.

Consider $\begin{bmatrix} 1 & 3 & -1 \\ 0 & -7 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore \quad x + 3y - z = 0 \quad \text{——} \quad (1)$

$-7y + 6z = 0$

$3z = 0$

Since $3z = 0 \Rightarrow z = 0$

Put $z = 0$ in $-7y + 6z = 0$, we get

$-7y + 6 \times 0 = 0 \Rightarrow y = 0$

& put $y = 0$ & $z = 0$ in eqn (1), we get $x = 0$

Hence solution of given system is $(x, y, z) = (0, 0, 0)$

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(2) Find the values of k such that the system of equations

$$x + ky + 3z = 0$$
$$4x + 3y + kz = 0$$
$$2x + y + 2z = 0$$

Solution:- We write given system of equations in matrix form $AX = B$

$$\begin{bmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Consider augmented matrix $[A|B]$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & k & 3 & 0 \\ 4 & 3 & k & 0 \\ 2 & 1 & 2 & 0 \end{array}\right]$$

$$= \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{bmatrix} \xrightarrow[R_3 - 2R_1]{R_2 - 4R_1} \begin{array}{c} \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & k & 3 \\ 0 & 3-4k & k-12 \\ 0 & 1-2k & -4 \end{bmatrix} \begin{array}{l} \\ \times(1-2k) \\ \times(3-4k) \end{array}$$

$\downarrow (3-4k)R_3 - (1-2k)R_2$

$$\begin{bmatrix} 1 & k & 3 \\ 0 & 3-4k & k-12 \\ 0 & 0 & \underbrace{-4(3-4k)-(1-2k)(k-12)}_{0} \end{bmatrix}$$

$(3-4k)(1-2k) - (1-2k)(3-4k) = 0$

$-4(3-4k) - (1-2k)(k-12)$

For non-trivial solution

$$\rho[A|B] = \rho[A] < n = 3$$

$$\left[\begin{array}{ccc} 1 & k & 3 \\ 0 & 3-4k & k-12 \\ 0 & 0 & \underbrace{-4(3-4k)-(1-2k)(k-12)}_{0} \end{array}\right]$$

For non-trivial solution

$\rho[A|B] = \rho[A] < n = 3$

$\therefore -4(3-4k)-(1-2k)(k-12) = 0$

$\therefore -12+16k-[k-12-2k^2+24k] = 0$

$\therefore \cancel{-12}+16k-k\cancel{+12}+2k^2-24k = 0$

$2k^2-9k = 0$

$\Rightarrow k(2k-9) = 0$

$\Rightarrow k=0$ or $k = 9/2$ — Required Solution.

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For Competative Exam :-

$$|A| = \begin{vmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{vmatrix} = 1(6-k) - k(8-2k) + 3(4-6)$$

$$= \cancel{6} - k - 8k + 2k^2 - \cancel{6}$$

$$= 2k^2 - 9k$$

For non-trivial solution;

$$|A| = 0$$

$$\therefore 2k^2 - 9k = 0$$

$$\Rightarrow k(2k-9) = 0$$

$$\Rightarrow k = 0 \text{ or } 2k - 9 = 0$$

$$\Rightarrow k = 0 \text{ or } k = 9/2$$