Lecture No-1: Vector Space over R

Santosh Shivlal Dhamone

Assistant Professor in Mathematics
Art's Commerce and Science College,Onde
Tal:- Vikramgad, Dist:- Palghar

santosh2maths@gmail.com

September 28, 2021

Contents

Lecture 1: Vector Space over R

Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
Arts,Commerce & Science College,Onde
Tal. Vikramgad,Dist. Palghar (MS)-401605
(Affiliated to the University of Mumbai)
NAAC Accredited- Grade-C (CGPA-1.85)
ISO-9001:2015 Certified
Year of Establishment: 2002

Linear Algebra - I
Unit II : Vector Spaces over IR
Lecture 1

Santosh Shivlal Dhamone
Assistant Professor in Mathematics
Arts Commerce and Science College, Onde

Internal Composition :-

Let V be any set. Then the mapping $f : V \times V \longrightarrow V$ is said to be internal composition and it is also called vector addition.

Example: $f : R \times R \longrightarrow R$

$$f(a, b) = a * b$$

where $*$ is any binary operation.

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

External Composition :

Let V and F be any non-empty set. Then the mapping $f : V \times F \longrightarrow V$ is said to be external composition in V over F also called scalar multiplication.

Vector Space :-

Let $(F, +, \cdot)$ be a field. The elements of F will be called scalars.

Here we learn vector space over $\mathbb{R}$.

$F = \mathbb{R}$

A non-empty set V whose elements will be called vectors, under the operation of addition $(+)$ & scalar multiplication $(\cdot)$ is said to be vector space over R, if it satisfies the following axioms.

Lecture 1: Vector Space over R

(i) There is defined an internal composition in V called addition (+) of vectors & denoted by '+'. Also for this composition V is an abelian group.

Axiom 1 : Closure w.r.t. Addition (+)

$u+v \in V$ for all $u, v \in V$

Axiom 2 : Commutativity w.r.t. addition (+)

$u+v = v+u$ for all $u, v \in V$

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Axiom 3: Associtivity w.r.t. addition (+)

$u+(v+w)=(u+v)+w$ for all $u,v,w \in V$.

Axiom 4: Existance of zero '0' in V.

There exist an element $0 \in V$ such that

$u+0=u=0+u$ for all $u \in V$.

This element $0 \in V$ will be called the zero vector & is said to be additive identity of V.

Axiom 5 :- Existance of additive inverse for each member of V

For every u in V, there is correspond-ing v in V, such that

$u + v = v + u = 0$

i.e. there exist a vector $-u \in V$ such that $u + (-u) = (-u) + u = 0$.

Lecture 1: Vector Space over R

My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

(ix) There is an external composition in V over R called scalar multiplication & denoted multiplicatively i.e. $px \in V$ for all $x \in V$ & $p \in R$.
In other words V is called closed w.r.t. scalar multiplication.

Axiom 6: Closure w.r.t. scalar multiplication
$px \in V$ for all $x \in V$ & $p \in R$.

Lecture 1: Vector Space over R

My Inspiration Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Axiom 7: Behaviour of 1 from R w.r.t elements in V.

The number 1 from R satisfies the following

$$1 \cdot u = u \quad \forall \; u \in V.$$

Axiom 8: Associtivity w.r.t. multiplication $(\cdot)$

$$p \cdot (qu) = (pq) \cdot u \quad \forall \; p, q \in R \;\&\; u \in V.$$

Axiom 9: Distributivity of $\cdot$ over $+$

For all $p, \cancel{q} \in R$ & $u, v \in V$

$$p(u+v) = pu + pv \text{ in } V.$$

Axiom 10: Distributivity of $+$ over $\cdot$

For all $p, q \in R$ & $u \in V$

$$(p+q)u = pu + qu \text{ in } V.$$