Lecture No-12: System of Linear Equations and Matrices

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My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

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Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's

Arts,Commerce & Science College,Onde

Tal. Vikramgad,Dist. Palghar (MS)-401605

(Affiliated to the University of Mumbai)

NAAC Accrediated- Grade-C (CGPA-1.85)

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Linear Algebra - I

Unit I : System of Linear Equations and Matrices

Lecture - 12

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Lecture 12: System of Linear Equations and Matrices

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Goals

- Define Elementary Matrices, corresponding to elementary operations.
- We will see that performing an elementary row operation on a matrix A is same as multiplying A on the left by an elmentary matrix E.
- We will see that any matrix A is invertible if and only if it is the product of elementary matrices.

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Definition

Definition: A square matrix A (of size $n \times n$) is called an **Elementary Matrix** if it can be obtained from the identity matrix I_n by a single elementary row operation. That means A is obtained by

- switching two rows on I_n, or
- multiplying a row of I_n by a scalar $c \neq 0$ or
- adding a scalar multiple of a row of I_n to another row.

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Inverse of Elementary Matrices

Theorem If E is elementary, then E^{-1} exists and is elementary.

- **Proof** For each of the three types of elementary matrices, write down the inverse and check. I will do it on the board.

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Example 2.4.1

Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Is this matrix elementary. If yes why?
Answer: Yes, it is. The matrix A is obtained from I_3 by adding 3 time the first row of I_3 to the second row.

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Example 2.4.2

Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

Is this matrix elementary. If yes why?
Answer: Yes, it is. The matrix A is obtained from I_3 by multiplying its third row by 1.5.

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Example 2.4.3

Let

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Is this matrix elementary. If yes why?
Answer: Yes, it is. The matrix A is obtained from I_3 by switching its first and third row.