

# Art's Commerce and Science College, Onde Tal:- Vikramgad, Dist:- Palghar

My Inspiration Shri. V.G. Patil Saheb Dr. V. S. Sonawne

Santosh Shivl Dhamone

## Lecture No-14: System of Linear Equations and Matrices

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Linear Algebra - I

Unit I: System of Linear Equations and Matrices



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#### Theorem 2.2

**Theorem.** A square matrix *A* is invertible if and only if it is product of elementary matrices.

**Proof.** Need to prove two statements. First prove, if A is product it of elementary matrices, then A is invertible. So, suppose  $A=E_kE_{k-1}\cdots E_2E_1$  where  $E_i$  are elementary. Since elementary matrices are invertible,  $E_i^{-1}$  exists. Write  $B=E_i^{-1}E_i^{-1}\cdots E_i^{-1}, E_i^{-1}$ . Then

$$AB = (E_k E_{k-1} \cdots E_2 E_1)(E_1^{-1} E_2^{-1} \cdots E_{k-1}^{-1} E_k^{-1}) = I.$$

Similarly, BA = I. So, B is the inverse of A.

Matrices: \$2.4 Elementary Matrices

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Let

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{I}_{3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find its inverse, using the theorem above.

**Solution.** The method is to reduce A to  $I_3$  by elementary operations, and interpret it in terms of multiplication by elementary matrices.

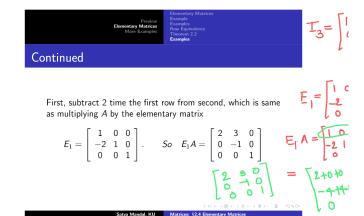
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Matrices: \$2.4 Elementary Matrices



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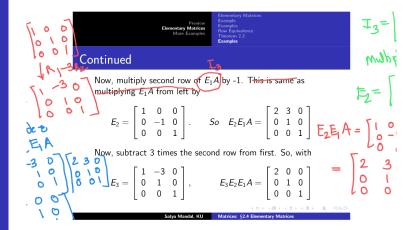
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#### Continued

Now, multiply the first row of  $E_3E_2E_1A$  by .5. So, with

$$E_4 = \left[ \begin{array}{ccc} .5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right],$$

$$E_{4} = \begin{bmatrix} .5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad E_{4}E_{3}E_{2}E_{1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1}$$

If you wish, you can write it more explicitly, by expanding the right hand side.