My Inspiration
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Lecture No-14: System of Linear Equations and Matrices

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Linear Algebra - I

Unit I : System of Linear Equations and Matrices

Lecture - 14

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Lecture 14: System of Linear Equations and Matrices

Theorem 2.2

Theorem. A square matrix A is invertible if and only if it is product of elementary matrices.

Proof. Need to prove two statements. First prove, if A is product it of elementary matrices, then A is invertible. So, suppose $A = E_k E_{k-1} \cdots E_2 E_1$ where E_i are elementary. Since elementary matrices are invertible, E_i^{-1} exists. Write $B = E_1^{-1} E_2^{-1} \cdots E_{k-1}^{-1} E_k^{-1}$. Then

$$AB = (E_k E_{k-1} \cdots E_2 E_1)(E_1^{-1} E_2^{-1} \cdots E_{k-1}^{-1} E_k^{-1}) = I.$$

Similarly, $BA = I$. So, B is the inverse of A.

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Example 2.4.7

Let

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find its inverse, using the theorem above.

Solution. The method is to reduce A to I_3 by elementary operations, and interpret it in terms of multiplication by elementary matrices.

$$A = E_K \cdot E_{K-1} \cdots E_3 E_2 E_1$$

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Continued

First, subtract 2 time the first row from second, which is same as multiplying A by the elementary matrix

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad So \quad E_1A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$I_3 = [1$

$E_1 = \begin{bmatrix} 1 & 0 \\ -2 & \\ 0 & \end{bmatrix}$

$E_1 \cdot A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 0 & \end{bmatrix}$

$\begin{bmatrix} 2 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+0+0 \\ -4+4+ \\ 0 \end{bmatrix}$

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Continued

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 3R_2} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, multiply second row of E_1A (I_3) by -1. ~~This is same as multiplying~~ E_1A from left by

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad So \quad E_2E_1A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, subtract 3 times the second row from first. So, with

$$E_3 = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad E_3E_2E_1A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E_1A

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$I_3 =$

$E_2 =$

$E_2E_1A =$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

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Now, multiply the first row of $E_3E_2E_1A$ by .5. So, with

$$E_4 = \begin{bmatrix} .5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad E_4E_3E_2E_1A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

So,

$$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$$

If you wish, you can write it more explicitly, by expanding the right hand side.