My Inspiration
Shri. V.G. Patil Saheb
Dr. V. S. Sonawne

Santosh Shivlal Dhamone

Lecture No-15: System of Linear Equations and Matrices

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September 26, 2021

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NAAC Accredited- Grade-C (CGPA-1.85)
ISO-9001:2015 Certified
Year of Establishment: 2002

Linear Algebra - I

Unit I : System of Linear Equations and Matrices

Lecture - 15

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Example 2.4.8

$$Let \quad A = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 5 & -3 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}$$

Find an elementary matrix so that $EA = C$.

Solution. If we add third row of A to its first row, we get C. Let E be the matrix that is obtained from the identity matrix I_3 by adding its third row to the first. Or

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad so \quad EA = C.$$

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Example 2.4.9

$$\text{Let} \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

Compute the inverse of A by elementary operations.

Solution. I_3 is obtained from A by adding -3 times second row of A to third row of A. Accordingly write

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \quad So, \ EA = I_3, \quad Check \quad AE = I_3.$$

So, $A^{-1} = E$.

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Example 2.4.10

$$Let \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 2 & 5 & 7 \end{bmatrix}$$

Find a sequence of elementary matrices whose product is A.

Solution. Let E_1 be the matrix obtained by subtracting the second row of I_3 from its third row and A_1 is obtained by the same operation on A. So,

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 0 & 0 & 1 \end{bmatrix}, \quad so \quad E_1A = A_1.$$

E_2 be the the matrix obtained by subtracting 2 times the first row of I_3 from its second row and A_2 is obtained by the same operation on A_1. So,

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad so \quad E_2 A_1 = A_2.$$

E_3 be the the matrix obtained by subtracting 2 times the second row of I_3 from its first row and A_3 is obtained by the same operation on A_2. So,

$$E_3 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad so \quad E_3 A_2 = A_3.$$

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E_4 be the the matrix obtained by subtracting 3 times the third row of I_3 from its first row and A_4 is obtained by the same operation on A_3. So,

$$E_4 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3, \quad so\ E_4A_3 = A_4 = I_3.$$

Therefore

$$E_4E_3E_2E_1A = I_3 \quad and \quad A^{-1} = E_4E_3E_2E_4.$$

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$$A^{-1} = E_4E_3E_2E_1 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E_2E_1$$

$$= \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E_2E_1 = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E_1$$

$$= \begin{bmatrix} 5 & -2 & -3 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E_1 = \begin{bmatrix} 5 & -2 & -3 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 & -3 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

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