QP Code : 24900

(2 1/2 Hours) [Total Marks: 75

N.B.: (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective subquestions.

1. (a) Attempt any *one* of the following: [8]
   (i) Show that the rate of convergence of the secant method is $\frac{1}{2}(1\pm\sqrt{5})$.
   (ii) Show that the Newton – Raphson iterative formula applied to the function $f(x)= x^2 - a$, $a > 0$ leads to the iterative formula
   $$x_{k+1} = \frac{1}{2}(x_k + \frac{a}{x_k}),\ x_0 > 0$$
   for evaluating $\sqrt{a}$. Also for the function $f(x)= x^p - a$, show that the sequence given by,
   $x_{k+1} = \frac{1}{p}((p-1)x_k + \frac{a}{x_k^{p-1}})$, $x_0 > 0$ can be used to evaluate $a^{\frac{1}{p}}$.

   (b) Attempt any *three* of the following: [12]
   (i) Find the number of terms n to be taken in the expansion of e^x correct to 6 places of decimals, when x = 1.
   (ii) Determine the iterative formula to find $\sqrt{N}$ where N is a positive integer, using Newton – Raphson method.
   (iii) Taking $x_0 = 0$ and $x_1 = 1$, solve by Regula- Falsi method the equation $x - \cos x = 0$. Perform two iterations.
   (iv) Find a negative root of $x^3 - 2x + 5 = 0$, using Fixed point iterative method. Perform two iterations.

2. (a) Attempt any *one* of the following: [8]
   (i) If p_k is an approximation of the root p of the polynomial equation $P_n(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0$, then show that the next approximation to the root using Birge-Vieta method is $p_{k+1} = p_k - \frac{b_n}{c_{n-1}}$, k = 0, 1, 2... where b_k satisfies the recurrence relation $b_k = a_k + p\, b_{k-1}$ with $b_0 = a_0$ and c_k satisfies the recurrence relation $c_k = b_k + pc_{k-1}$ and $c_0 = b_0$.
   (ii) Let A be the coefficient matrix of the system of equations. Explain how to decompose A into the product of a lower triangular matrix L and upper triangular matrix U using Triangularization method.

   (b) Attempt any *three* of the following: [12]
   (i) Using synthetic division find the value of P(2), P′(2) for the polynomial
   $$x^5 - 2x^4 + 4x^3 - x^2 - 7x + 5 = 0.$$
   (ii) Using Strum's sequence obtain the exact number of real root and complex roots of the polynomial $x^3 + x - 1 = 0$.
   (iii) Decompose A given below into lower triangular matices L and L^T such that $A = LL^T$ using Cholesky's method.
   $$A = \begin{bmatrix} 9 & 1 & 0 \\ 1 & 9 & 1 \\ 0 & 1 & 9 \end{bmatrix}$$
   (iv) The system of equations AX =b is to be solved iteratively by $X^{(k+1)} = M\,X^{(k)} + b$, where $M = -D^{-1}(L + U)$ and D is the identity matrix. Suppose $A = \begin{bmatrix} 1 & k \\ 3k & 1 \end{bmatrix}$, where $k \neq \sqrt{3}/3$, k real.

[TURN OVER

Find necessary and sufficient condition on k for convergence of Jacobi iterative method.

3. (a) Attempt any *one* of the following: **[8]**

(i) Let A be a real symmetric matrix. Using Jacobi method reduce A to a diagonal matrix by a series of orthogonal transformations $S_1, S_2, \dots$ in 2×2 subspaces. Let $|a_{ik}|$ be the numerically largest off diagonal element of the matrix S_1^* such that $S_1^* A S^*$ is diagonalized and hence show that $\tan 2\theta = \frac{2a_{ik}}{a_{ii} - a_{kk}}, -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

(ii) Define condition number of a matrix A. Let $A(\propto) = \begin{bmatrix} 0.1\propto & 0.1\propto \\ 1.00 & 1.5 \end{bmatrix}$. Determine α such that cond($A(\alpha)$) is minimized. Use maximum absolute row sum norm.

(b) Attempt any *three* of the following: **[12]**

(i) Let A be a symmetric matrix given below. Apply Jacobi method to find orthogonal transformations S_1 and S_2.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ -1 & 2 & 1 \end{bmatrix}$$

(ii) Obtain first iteration matrix (A_2) and determine eigen value for $A = \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix}$ using Rutishauser method.

(iii) Find the largest eigen value and eigen vector in magnitude of the following matrix using Power method at the end of second iteration.

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 3 \end{bmatrix}$$

(iv) Find the smallest eigen value in magnitude of the matrix A using inverse power method at the end of second iteration. Also obtain corresponding eigen vector.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

Take initial approximation as $[1\ 1\ 1]^T$ near 3.

4. Attempt any *three* of the following: **[15]**

(i) Find a root by Secant method correct upto 4 decimal places for the equation $e^{-x} = \sin x$

(ii) Find the inverse of the following matrix using LU decomposition method. Take $l_{11} = l_{22} = l_{33} = 1$.

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

(iii) Find approximately the eigenvalues of the following matrix using Rutishauser method. Apply the procedure until the elements of the lower triangular part are less than 0.005 in magnitude.

$$\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$

(iv) Perform one iteration with Muller method for the equation $x^3 - (1/2) = 0$, $x_0 = 0$, $x_1 = 1$, $x_2 = (1/2)$.

(v) Solve the following system of equations using Jacobi iterative method. Do one iteration. $4x_1 + x_2 + x_3 = 2$, $x_1 + 5x_2 + 2x_3 = -6$, $x_1 + 2x_2 + 3x_3 = -4$. Take the initial approximation as $x^{(0)} = [0.5, -0.5, -0.5]^T$.

(vi) Find the largest eigen value in modulus and the corresponding eigen vector of the matrix A using power method at the end of second iteration.

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}.$$

Take initial vector as $[1\ 1\ 1]^T$.

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