Practical No 3: System of Linear Equations and Matrices

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Sanjeevan Gramin Vidyakiya & Samajik Sahayata Pratishthan's
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Linear Algebra- I

Unit I: System of Equations, Matrices

Practical No 3

Methods of Solving Non-Homogenous System: Gaussian Elimination Method: AX=B

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Methods of Solving Non-Homogenous System :-

Gauss Elimination Method : $AX = B$, where $B \neq 0$

(1) If $\rho(A|B) = \rho(A)$ = number of unknown (n) then;
System is consistant with unique solution.

(2) If $\rho(A|B) = \rho(A) <$ number of unknown (n), then;
system is consistant with infinite solutions
& in that case $(n-r)$ variables are assigned arbitrary values ; where n = no. of unknown & r = rank

(3) If $\rho(A|B) \neq \rho(A)$ then system is inconsistant & has no solution.

Note :-

(1) Try to convert augumeted matrix into an row echelon form using elementry row operations only.

(2) Then find values of unknown by using backward substitution method.

Examples :-

(1) Test for consistancy and solve following system of equation

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

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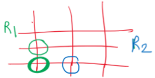
Solution :- We write given system of equations in Matrix form

$$AX = B$$

$$\therefore \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$



Consider

$$[A \mid B] = \begin{matrix} 3\times R_1 \\ -2\times R_2 \\ R_3 \end{matrix} \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$$\Big\downarrow \begin{matrix} -2R_2 + 3R_1 \\ R_3 - R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & -11 & 27 & 11 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$$\begin{array}{r} -26 \\ +15 \\ \hline \end{array}$$

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$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & -11 & 27 & 11 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$$\downarrow R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & -11 & 27 & 11 \\ 0 & 0 & 0 & 49 \end{array}\right]$$

$\rho(A) = 2$

Here $\rho(A|B) = 3$, $\rho(A) = 2$

$\therefore \rho(A|B) \neq \rho(A)$

In this case system has no solution.

Hence given system has no solution.

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(2) Test for consistancy & solve the following system of equations:-

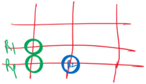
$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

Solution:- We write given system of equations in matrix form

$$AX = B$$

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

(a_{21}, a_{31}, a_{32})



We consider,

$$[A|B] = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$-7 \times$, $-3 \times$, $5 \times$, $5 \times$

$\downarrow$ $5R_2 - 3R_1$, $5R_3 - 7R_1$

−15	−9	−21	−12
15	130	10	45
0	124	−11	33

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$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 124 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$$

$\downarrow 11R_3 + R_2$

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 124 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A|B) = 2$

Here $\rho(A) = 2$, $\rho(A|B) = 2$ & $n =$ no. of unknown $= 3$

$\therefore \rho(A) = \rho(A|B) < n = 3$

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Put $y = \frac{t+3}{11}$ & $z = t$ in eqn (*)

$$5x + 3\left(\frac{t+3}{11}\right) + 7t = 4$$

$$\therefore \; 5x + \frac{3t+9}{11} = 4 - 7t$$

$$5x + \frac{3t+9}{11} = 4 - 7t$$

$$55x + 3t + 9 = 44 - 77t$$

$$55x = 44 - 77t - 3t - 9$$

$$\boxed{x = \frac{33 - 80t}{55}}$$

$\therefore \; x = \frac{33-80t}{55}$, $y = \frac{t+3}{11}$ & $z = t$ is set of solution

Thank You

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Examples :-

(1) Show that the system of equations

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$
$$2x - 3y - z = 5$$

are consistant & hence solve it.

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Solution :- The given system of equation, write in matrix form $AX = B$ as follows:

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix}$$

Consider,

$$[A|B] = \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right] \xrightarrow[3R_4 - 2R_1]{3R_2 - R_1} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \left[\begin{array}{ccc} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 10 & 3 \\ 0 & -15 & -7 \end{array}\right.$$

$3R_3 -$

$3R_4 +$

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Consider,

$$[A|B] = \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix}\left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right] \xrightarrow[3R_4 - 2R_1]{3R_2 - R_1} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix}\left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 10 & 3 & -2 \\ 0 & -15 & -7 & 13 \end{array}\right]$$

$$\xrightarrow[3R_4 + 15R_2]{3R_3 - 10R_2} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & -51 & 204 \end{array}\right]$$

$$\xrightarrow{29R_4 + 51R_3} \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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Here $\rho(A|B) = 3$, $\rho(A) = 3$, no. of unknown $= 3$

$\therefore \rho(A) = \rho(A|B) = n = 3$

Hence given system is consistent & has unique solution.

$$\therefore \begin{bmatrix} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & 29 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \\ -116 \\ 0 \end{bmatrix}$$

By back substitution method,

$$29z = -116 \Rightarrow z = \frac{-116}{29} = -4$$

$$3y - 2z = 11$$

$$\therefore 3y - 2 \times -4 = 11 \Rightarrow 3y = 11 \oplus 8 =$$

$$3y + 8 = 11 \Rightarrow 3y = 11 - 8 = 3 \Rightarrow y = 3/3 = 1$$

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$3y - 2z = 11$

$\therefore 3y - 2\times -4 = 11 \Rightarrow 3y = 11 - 8 =$

$3y + 8 = 11 \Rightarrow 3y = 11 - 8 = 3 \Rightarrow y = 3/3 = 1$

$3x + 3y + 2z = 1 \Rightarrow 3x + 3\times 1 + 2\times -4 = 1$

$\Rightarrow 3x + 3 - 8 = 1$

$\therefore 3x - 5 = 1$

$3x = 6$

$x = 6/3$

$\therefore x = 2$

Required solution of given system of equations is

$x = 2,\ y = 1\ \&\ z = -4.$

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(2) Find for what values of λ & μ, the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

Have (i) no solution

(ii) A unique solution

(iii) infinite number of solution.

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Solution:- We write given system of equations in matrix form

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

Consider, augmented matrix $[A|B]$.

$$[A|B] = \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$$\downarrow \; R_2 - R_1, \; R_3 - R_1$$

$$\begin{matrix} \\ R_2 \\ R_3 \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

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$$\begin{array}{l} \\ R_2 \\ R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$$\Big\downarrow R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

(i) No solution.

$$\rho(A) \neq \rho(A|B)$$

$$2 \neq 3$$

$$\therefore \quad \lambda - 3 = 0 \Rightarrow \lambda = 3$$

$$\& \quad \mu - 10 \neq 0 \Rightarrow \mu \neq 10$$

$$\therefore \quad \mu \neq 10 \ \& \ \lambda = 3$$

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(ii) A unique soln.

If system of equation has unique solution

$\therefore \rho(A) = \rho(A|B) =$ no. of unknown.

$\therefore$ In this case $\lambda - 3 \neq 0$ & $\mu - 10 \neq 0$

$\therefore \lambda \neq 3$ & $\mu \neq 10$.

(iii) infinite number of solution.

We know that system of equation has infinite solution when $\rho(A) = \rho(A|B) <$ number of unknown

$\rho(A) = \rho(A|B) = 2 < 3$

$\therefore \lambda - 3 = 0$ & $\mu - 10 = 0$

$\therefore \lambda = 3$ & $\mu = 10$.

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Non-Homogenous System of linear equations :-

Let $AX = B$ be a system of linear equations in matrix form. If $B \neq 0$ then given system of equations is non-homogenous.

(1) If $\rho(A|B) = \rho(A) = n$;

where $\rho(A|B)$ — length of augumented matrix $[A|B]$
$\rho(A)$ — length of matrix A
n — number of unknown

then given system of non-homogenous linear equations is consistant & has unique solution.

(2) If $\rho(A|B) = \rho(A) < n$ then system is consistant but it has infinitely many solution.

(3) If $\rho(A|B) \neq \rho(A)$ then system of non-homogenous linear equations is inconsistant & has no solution.

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Non-Homogenous System of linear equations :-

Let $AX = B$ be a system of linear equations in matrix form. If $B \neq 0$ then given system of equations is non-homogenous.

(1) If $\rho(A|B) = \rho(A) = n$;

where $\rho(A|B)$ — length of augumented matrix $[A|B]$

$\rho(A)$ — length of matrix A

n — number of unknown.

then given system of non-homogenous linear equations is consistent & has unique solution.

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(1) If $\rho(A|B) = \rho(A) = n$;

where $\rho(A|B)$ – length of augumented matrix $[A|B]$

$\rho(A)$ – length of matrix A

n – number of unknown.

then given system of non-homogenous linear equations is consistent & has unique solution.

(2) If $\rho(A|B) = \rho(A) < n$ then system is consistant but it has infinitely many solution.

(3) If $\rho(A|B) \neq \rho(A)$ then system of non-homogenous linear equations is inconsistant & has no solution.

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Examples :-

(5) For what values of λ, the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

have a solution & solve completely in each case.

Lecture 6: System of Linear Equations and Matrices

Sol^n :- Given system of equations is

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

We write given system of equations in matrix form

$$AX=B$$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

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Consider augmented matrix $[A|B]$

$$\therefore [A|B] = \begin{array}{c} R_1 \to \\ R_2 \to \\ R_3 \to \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

Applying Row operations on $[A|B]$

$$\downarrow R_2 - R_1, \quad R_3 - R_1$$

$$\begin{array}{c} \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 3 & 9 & \lambda^2 - 1 \end{array}\right]$$

$$\downarrow R_3 - 3R_2$$

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$R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 0 & 0 & (\lambda-1)(\lambda-2) \end{array}\right]$$

$(\lambda^2 - 1) - 3(\lambda - 1)$

$\Rightarrow (\lambda-1)(\lambda+1) - 3(\lambda-1)$

$\Rightarrow (\lambda-1)[\lambda+1-3]$

$\Rightarrow (\lambda-1)(\lambda-2)$

Case (1) :- If $\rho(A) = \rho(A|B) = n$ = no. of unknown then it has unique solution.

Here $\rho(A) = 2$ & $n = 3$

Hence $\rho(A) = 2 < n = 3$

So there does not exist unique solution

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Case (2) :- If $\rho(A|B) = \rho(A) < n$ then it has infinitely many solutions.

Here $\rho(A) = 2$, $n = 3$ & $\rho(A|B) = 3$

To make $\rho(A|B) = 2$, we take or

$(\lambda-1)(\lambda-2) = 0$

$\Rightarrow \lambda - 1 = 0$ or $\lambda - 2 = 0$

= = or = 2

$\therefore$ Given system has infinetly many solutions when $\lambda = 1$, $\lambda = 2$

Lecture 6: System of Linear Equations and Matrices

Case (3) :- No Solution

If $\rho(A) \neq \rho(A|B)$ then system of equation has no solution.

Here $\rho(A) = 2$ & $\rho(A|B) = 3$

when $(\lambda-1)(\lambda-2) \neq 0$

$\Rightarrow \lambda \neq 1$ & $\lambda \neq 2$

$\therefore$ When $\lambda \neq 1, 2$ then given system has no solution

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(6) Investigate the values of λ & μ so that the equations:

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have (i) no solution

(ii) a unique solution

(iii) an infinite number of solution.

Solution :- Write given system of equations in matrix form $AX = B$ as follows

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 4 \end{bmatrix}$$

Consider Augmented matrix $[A|B]$ & apply row operations on it.

To make first entry in row second & third (i.e. $a_{21} = 0$ & $a_{31} = 0$) is equal to zero using firs

Consider

$$[A|B] = \begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \\ R_3 \rightarrow \end{array} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & 2 & 4 \end{array}\right]$$

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Consider

$$[A \mid B] = \begin{array}{c} R_1 \to \\ R_2 \to \\ R_3 \to \end{array} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$\Big\downarrow \begin{array}{l} 2R_2 - 7R_1 \\ R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array}\right]$$

(ii) No solution :-

If $\rho(A|B) \neq \rho(A)$ then system of equation has no solut

Here $\rho(A|B) = 3$ & $\rho(A) = 3$

$\therefore$ If $\mu - 9 = 0 \Rightarrow \mu = 9$ then $\rho(A|B) = 2$

& $\rho(A) = 2$ when $\lambda - 5 = 0 \Rightarrow \lambda = 5$.

Hence if $\lambda = 5$ then given system has no solutio

because $\rho(A) = 2 \neq \rho(A|B) = 3$

(ii) a unique solution :-

For unique solution

$$\rho(A|B) = \rho(A) = n$$

Here $n = 3$

$$\therefore \rho(A|B) = \rho(A)$$

$$\Rightarrow \lambda - 5 \neq 0 \;\&\; \mu - 9 \neq 0$$

$$\Rightarrow \lambda \neq 5 \;\&\; \mu \neq 9$$

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(iii) an infinite solution

For an infinite solution

$$\rho(A|B) = \rho(A) < n$$

Here $n = 3$

$\therefore \quad \rho(A|B) = \rho(A) = 2$

$\Rightarrow \quad \lambda - 5 = 0 \quad \& \quad \mu - 9 = 0$

$\Rightarrow \quad \lambda = 5 \quad \& \quad \mu = 9$

For $\lambda = 5 \qquad \rho(A) = 2$

$\& \quad \mu = 9 \qquad \rho(A|B) = 2.$

$\therefore$ For an infinite solution $\mu = 9$ & $\lambda = 5$.