Lecture No-1: Linear Transformation

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Lecture 1: Linear Transformation

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USMT 402 || Linear Algebra II

Unit I : Linear Transformation

Lecture 1

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Lecture 1: Linear Transformation

In first year, we studied a function.
function is a mapping from set A to set B

$$f : A \longrightarrow B$$

$$f : x \longrightarrow x^3 = y$$

$$f(x) = x^3 \quad \text{or} \quad y = x^3$$

$\therefore$ For $x \in A$, we get $y = f(x) \in B$.

e.g. $f : x \longrightarrow x^3 = y$

A   B

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Illustration 1 :-

Suppose $U = V = R^2 = R \times R$ Vector spaces then we can define a transformation

$$T : U \longrightarrow V$$

i.e. $T : R^2 \longrightarrow R^2$ as $(x, y) \in R^2$

$$T(x, y) = (x+y, x)$$

Then following will be images of some elements:

$$T(1, 0) = (1+0, 1) = (1, 1)$$
$$T(2, 3) = (2+3, 2) = (5, 2)$$
$$T(0, 1) = (0+1, 0) = (1, 0)$$
$$T(-1, -1) = (-1-1, -1) = (-2, -1)$$

Illustration 2:-

$T : R^2 \longrightarrow R^3$ defined by

$T(x, y) = (x, y, 2x)$

Now, we calculate image of some points in R^2.

$T(0, 0) = (0, 0, 0)$

$T(1, 1) = (1, 1, 2)$

$T(-1, 1) = (-1, 1, -2)$

$T(2, 3) = (2, 3, 4)$ so on.

Defination & Properties of Linear Transformation :-

Defination :- Let V & U be two vector spaces over $\mathbb{R}$. Then a transformation $T : V \longrightarrow U$ is called a linear transformation if

(i) $T(x+y) = T(x) + T(y) \quad \forall\, x, y \in V$

(ii) $T(cx) = c\,T(x) \quad \forall\, x \in V \;\&\; c \in \mathbb{R}$

Illustration 3:

Let $T : R^2 \longrightarrow R$ be defined by

$$T(x, y) = x + y \quad , \forall (x, y) \in R^2$$

To check whether T is a linear transformation.

Soln:- To prove that

(i) $T(X+Y) = T(X) + T(Y)$; $X, Y \in R^2$

(ii) $T(c \cdot X) = c\,T(X)$; $X \in R^2, c \in R$

(i) Let $X, Y \in R^2$ then $X = (x_1, y_1)$, $Y = (x_2, y_2)$

T.P.T. $T(X+Y) = T(X) + T(Y)$

Consider

$$L.H.S = T(X+Y)$$

Lecture 1: Linear Transformation

$$
\begin{aligned}
L.H.S &= T(X+Y) \\
&= T((x_1, y_1) + (x_2, y_2)) \\
&= T((x_1+x_2, y_1+y_2)) \\
&= (x_1+x_2) + (y_1+y_2) \quad (\because T(x,y) = x+y) \quad \text{——— (1)}
\end{aligned}
$$

Now, consider,

$$
\begin{aligned}
R.H.S &= T(X) + T(Y) \\
&= T(x_1, y_1) + T(x_2, y_2) \quad (\because T(x,y) = x+y) \\
&= x_1 + y_1 + x_2 + y_2 \\
&= (x_1+x_2) + (y_1+y_2) \quad \text{——— (2)}
\end{aligned}
$$

By equations (1) & (2), we get

$$
\begin{aligned}
L.H.S &= R.H.S \\
T(X+Y) &= T(X) + T(Y)
\end{aligned}
$$

Now, we have T.P.T $T(c\cdot X) = c\cdot T(X)$

(ii) Consider,

$$
\begin{aligned}
L.H.S &= T(c\cdot X) \\
&= T(c\cdot(x_1, y_1)) \\
&= T(cx_1, cy_1) \\
&= cx_1 + cy_1 \qquad \left[\because T(x,y) = x+y\right] \\
&= c(x_1+y_1) \\
&= c\,T(X) \\
&= R.H.S
\end{aligned}
$$

Hence property (i) & (ii) is proved.

Hence $T: R^2 \longrightarrow R$ defined by $T(x,y) = x+y$ is linear transfor