



# Art's Commerce and Science College, Onda

Tal:- Vikramgad, Dist:- Palghar

## *Linear Algebra-II*

My Inspiration  
Shri. V.G. Patil  
Saheb  
Dr. V. S.  
Sonawne

Santosh Shival  
Dhamone

## Lecture No-1: Linear Transformation

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January 12, 2022



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## Definition of Linear Transformation

100% HUMAN RESOURCES AND 0% AI



Santosh Dhamone  
Mathematics Professor

# Lecture 1: Linear Transformation

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(Affiliated to the University of Mumbai, Mumbai)  
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## USMT 402 || Linear Algebra II

### Unit I : Linear Transformation Lecture 1



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# Lecture 1: Linear Transformation

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In first year, we studied a function.  
function is a mapping from set A to set B

$$f : A \longrightarrow B$$

$$f : x \longrightarrow x^3 = y$$

$$f(x) = x^3 \text{ or } y = x^3$$

$\therefore$  for  $x \in A$ , we get  $y = f(x) \in B$ .





# Lecture 1: Linear Transformation

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Illustration 1:-

Suppose  $u = v = \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$  Vector spaces then we can define a transformation

$$T: U \longrightarrow V$$

i.e.  $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$  as  $(x, y) \in \mathbb{R}^2$

$$T(x, y) = (x+y, x)$$

Then following will be images of some elements:

$$T(1, 0) = (1+0, 1) = (1, 1)$$

$$T(2, 3) = (2+3, 2) = (5, 2)$$

$$T(0, 1) = (0+1, 0) = (1, 0)$$

$$T(-1, -1) = (-1-1, -1) = (-2, -1)$$



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Illustration 2:-

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  defined by

$$T(x, y) = (x, y, 2x)$$

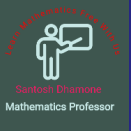
Now, we calculate image of some points in  $\mathbb{R}^2$ .

$$T(0, 0) = (0, 0, 0)$$

$$T(1, 1) = (1, 1, 2)$$

$$T(-1, 1) = (-1, 1, -2)$$

$$T(2, 3) = (2, 3, 4) \text{ so on.}$$



# Lecture 1: Linear Transformation

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**Definition & Properties of Linear Transformation:-**

**Definition:-** Let  $V$  &  $U$  be two vector spaces over  $R$ . Then a transformation  $T: V \rightarrow U$  is called a linear transformation if

$$(i) \quad T(x+y) = T(x) + T(y) \quad \forall x, y \in V$$

$$(ii) \quad T(cx) = cT(x) \quad \forall x \in V \quad \forall c \in R$$

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## Illustration 3:

Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$T(x, y) = x + y \quad \forall (x, y) \in \mathbb{R}^2$$

To check whether  $T$  is a linear transformation.

Sol<sup>n</sup>:- To prove that

$$(i) T(X+Y) = T(X) + T(Y) \quad ; \quad X, Y \in \mathbb{R}^2$$

$$(ii) T(c \cdot X) = c T(X) \quad ; \quad X \in \mathbb{R}^2, c \in \mathbb{R}$$

(i) Let  $X, Y \in \mathbb{R}^2$  then  $X = (x_1, y_1), Y = (x_2, y_2)$

$$\text{T.P.T. } T(X+Y) = T(X) + T(Y)$$

Consider

$$\text{L.H.S} = T(X+Y)$$



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$$\begin{aligned}
 \text{L.H.S} &= T(X+Y) \\
 &= T((x_1, y_1) + (x_2, y_2)) \\
 &= T((x_1+x_2, y_1+y_2)) \\
 &= (x_1+x_2) + (y_1+y_2) \quad (\because T(x, y) = x+y)
 \end{aligned}$$

Now, consider, ①

$$\begin{aligned}
 \text{R.H.S} &= T(X) + T(Y) \\
 &= T(x_1, y_1) + T(x_2, y_2) \quad (\because T(x, y) = x+y) \\
 &= x_1 + y_1 + x_2 + y_2 \\
 &= (x_1+x_2) + (y_1+y_2) \quad \text{--- ②}
 \end{aligned}$$

By equations ① & ②, we get

$$\frac{\text{L.H.S}}{T(X+Y)} = \frac{\text{R.H.S}}{T(X) + T(Y)}$$



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Now, we have T.P.T  $T(c \cdot X) = c \cdot T(X)$

(ii) Consider,

$$\begin{aligned}
 \text{L.H.S} &= T(c \cdot X) \\
 &= T(c \cdot (x_1, y_1)) \\
 &= T(cx_1, cy_1) \\
 &= cx_1 + cy_1 \\
 &= c(x_1 + y_1) \\
 &= c T(X) \\
 &= \text{R.H.S}
 \end{aligned}$$

[  $\because T(x, y) = x + y$  ]

Hence property (i) & (ii) is proved.

Hence  $T: \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $T(x, y) = x + y$  is linear transformation