Lecture No. 10: Module 1: Arithmetic, Algebra and Combinatorics

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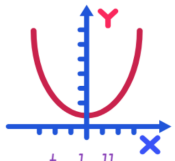

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Indian Mathematics

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Introduction:

Introduction:

- "Varga" means square, and "Prakriti" means nature or type.
- The Varga-Prakriti problem involves solving quadratic indeterminate equations of the form:
  $x^2 = Dy^2 + 1$
  or more commonly in its general form:
  $x^2 - Dy^2 = 1$
  This is known today as the **Pell's equation.**
- In ancient Indian mathematics, solving this equation was referred to as the *Varga-Prakriti* problem.

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

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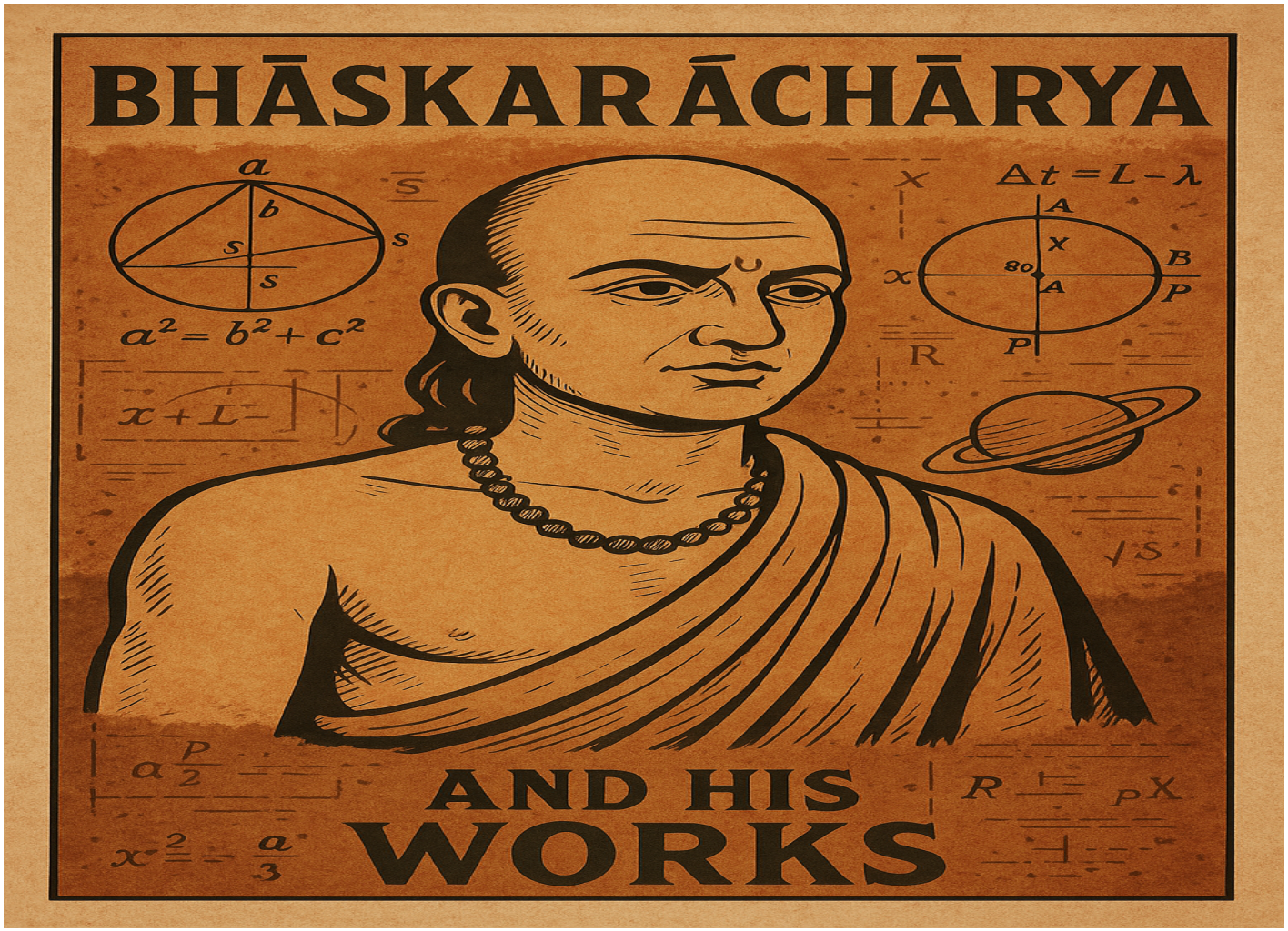

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Bhaskaracharya and His Work:

Bhaskaracharya and His Work

- The 12^{th}-century Indian mathematician Bhaskaracharya II discusses the Varga-Prakriti in his treatise Bijaganita (Algebra).
- He called this type of equation Varga-Prakriti and gave a general method for solving it.
- He adopted and improved upon the Chakravala method, originally developed by Jayadeva and earlier versions by Brahmagupta.

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

The Varga-Prakriti Problem:

General form:

$x^2 - Dy^2 = 1$
where D is a non-square positive integer.

Goal:

Find integer solutions x, y.

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Bhaskaracharya's Method – Chakravala Method:

Bhaskaracharya described a cyclic and efficient algorithm to solve the Varga-Prakriti equation, called the Chakravala (cyclic) method.

Steps of the Chakravala Method::

Step 1: Choose initial triple a, b, k such that:

$a^2 - Db^2 = k$

Begin with small integers such that this relation is true with small k.

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Step 2: Find m such that:

$\frac{a+bm}{k}$ is an integer
and m satisfies:
$m^2 \equiv D \ (mod \ \ k)$
Choose the value of m such that $|m^2 - D|$ is minimized.

Step 3: Update triple:

$$a' = \frac{am+Db}{|k|}, \ b' = \frac{a+bm}{|k|}, \ k' = \frac{m^2-D}{k}$$

Step 4: Repeat steps until $k = \pm 1$.

Step 5: The final pair (a, b) gives the solution to:

$$x^2 - Dy^2 = 1$$

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Solved Example:

Solving $x^2 - 61y^2 = 1$:

Let us follow the Chakravala steps (brief sketch):

- Start with $a = 8, b = 1, k = 8^2 - 61 \times 1^2 = 3$
- Try values of m such that $m^2 \equiv 61 \ (mod \ \ 3)$
- Continue cycle (around 5–6 steps)
- Final solution: x=1766319049, y=226153980

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Why Bhaskaracharya's Method Is Remarkable:

It provided integer solutions efficiently centuries before similar techniques in Europe.

It is algorithmic in nature — a clear step-by-step process.

European mathematicians (like Euler and Lagrange) only discovered the solution to Pell's equation much later.

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Connections with Modern Math:

Varga-Prakriti = Pell's equation.

Chakravala method = continued fraction algorithm in disguise.

Bhaskaracharya's insights laid the groundwork for later developments in Diophantine equations.

The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Summary:

Concept	Description
Varga-Prakriti	Equation of the form $x^2 - Dy^2 = 1$
Key Contributor	Bhaskaracharya II
Main Method	Chakravala (cyclic) method
Objective	Find integer solutions for non-square D
Significance	Ancient algorithm rivaling modern algebr