# Lecture No. 11: Module 1: Arithmetic, Algebra and Combinatorics

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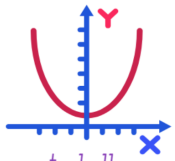

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## Indian Mathematics

1. The Zero and the Decimal System: The early appearance of Zero
2. Terms for the multiples of ten like 10, 20, 30 etc. in Rigveda. Terms for the higher powers of 10, given by Aryabhat, Mahaviracharya and Bhaskaracharya
3. The elementary operations like addition, subtraction, multiplication, division. Operations with fractions. Operations with zero. Squares and Cubes.
4. Methods to Obtain Square Roots and Cube Roots in Indian Mathematics
5. Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

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## Indian Mathematics

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## Indian Mathematics

# Step-by-Step Solution Using Chakravala Method



## Find integer solutions to $x^2 13y^2 = 1$:

## i.e., solve: $x^2 = 13y^2 + 1$:

## Step 1: Choose initial triple $(a, b, k)$

We look for small integers $a, b$ such that
$a^2 13b^2 = k$
Try:
Let $a = 7, b = 2$
$7^2 13 \times 2^2 = 4952 = 3$
$\implies (a, b, k) = (7, 2, 3)$

# Step-by-Step Solution Using Chakravala Method

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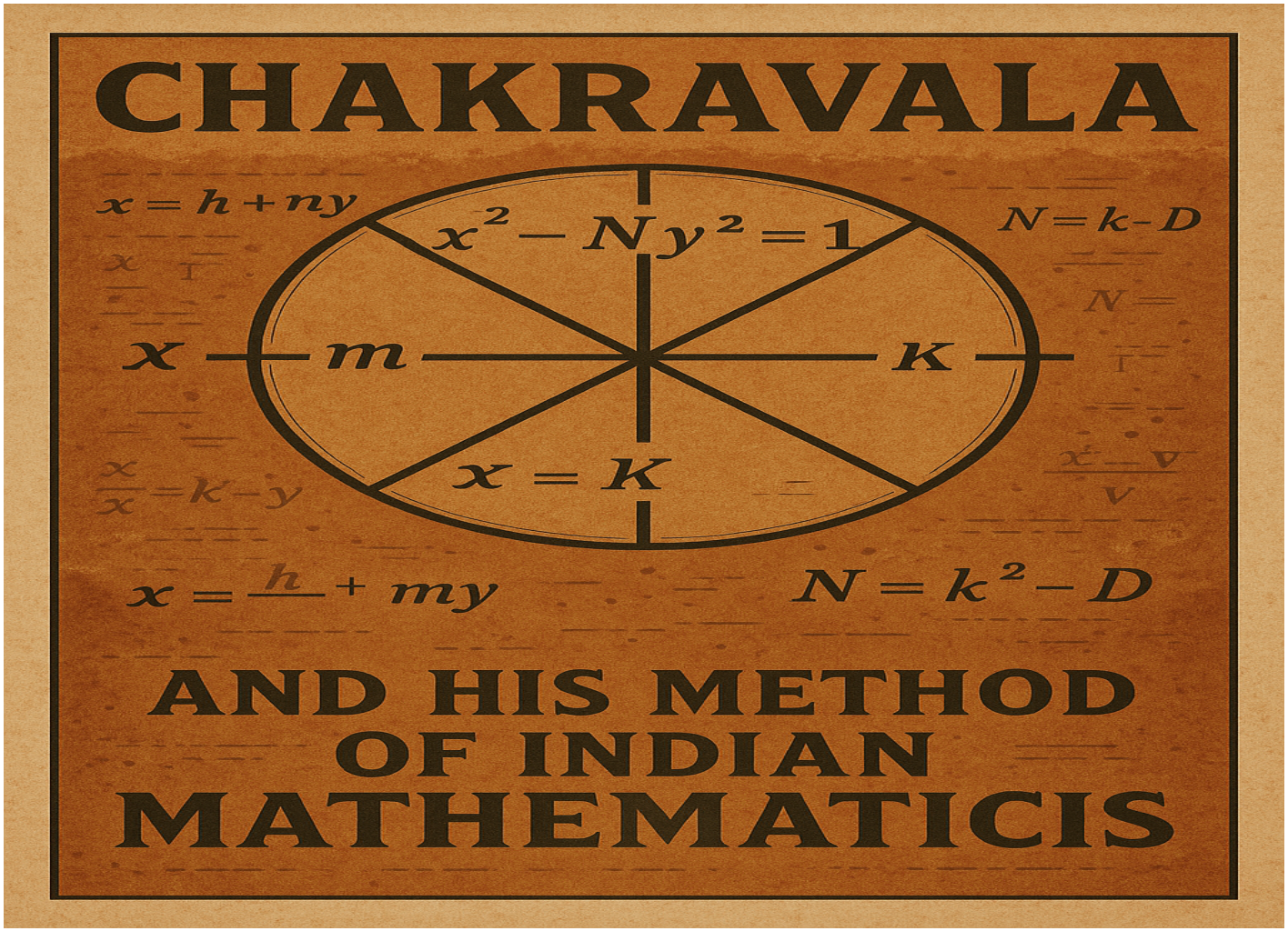

# Step-by-Step Solution Using Chakravala Method



## Step 2: Find integer $m$ such that

- $(a + bm)$ divisible by $k$, and
- $m^2 \equiv 13 \ (mod \ \ k)$
- Minimize $|m^2 - 13|$

Here $k = -3$, so:
Try $m = 1$;
$7 + 21 = 9, 9(3) = 3 \implies$ *divisible*
so, take m=1

# Step-by-Step Solution Using Chakravala Method



## Step 3: Compute next values

Use formulas:

$$a' = \frac{am + 13b}{|k|}, b' = \frac{a + bm}{|k|}, k' = \frac{m^2 - 13}{k}$$

Now plug in: $a = 7, b = 2,\ k = 3,\ m = 1$

$$a' = \frac{71 + 132}{|-3|} = \frac{7 + 26}{3} = \frac{33}{3} = 11$$

$$b' = \frac{7 + 21}{|-3|} = \frac{7 + 2}{3} = \frac{9}{3} = 3$$

$$k' = \frac{1^2 - 13}{-3} = \frac{1 - 13}{-3} = \frac{-12}{-3} = 4$$

# Step-by-Step Solution Using Chakravala Method



## New triple: $(a, b, k)=(11, 3, 4)$

## Step 4: Repeat with new triple

Now: $(a = 11, b = 3, k = 4)$

Find $m$ such that:

$11 + 3m \equiv 0 \mod 4 \rightarrow 3m \equiv -11 \equiv 1 \mod 4$

so $3m \equiv 1 \mod 4 \implies m \equiv 3 \mod 4$

Try $m = 3$:

Check $|m^2 - 13| = |9 - 13| = 4$

Okay, take $m = 3$

# Step-by-Step Solution Using Chakravala Method



## New triple: $(a, b, k)=(11, 3, 4)$

## Step 4: Repeat with new triple

Now: $(a = 11, b = 3, k = 4)$

Find $m$ such that:

$11 + 3m \equiv 0 \mod 4 \rightarrow 3m \equiv -11 \equiv 1 \mod 4$

so $3m \equiv 1 \mod 4 \implies m \equiv 3 \mod 4$

Try $m = 3$:

Check $|m^2 - 13| = |9 - 13| = 4$

Okay, take $m = 3$

# Step-by-Step Solution Using Chakravala Method



## Step 5: Compute new triple

$$a' = \frac{11.3 + 133}{4} = \frac{33 + 39}{4} = \frac{72}{4} = 18$$

$$b' = \frac{11 + 33}{4} = \frac{11 + 9}{4} = \frac{20}{4} = 5$$

$$k' = \frac{3^2 - 13}{4} = \frac{9 - 13}{4} = \frac{-4}{4} = -1$$

New triple: $(a, b, k) = (18, 5, -1)$

# Step-by-Step Solution Using Chakravala Method



## Final Step: We now have

$a^2 - 13b^2 = -1 \implies 18^2 - 13 \times 5^2 = 324 - 325 = -1$

To get a solution for: $x^2 - 13y^2 = 1$

Multiply the current solution with itself (composition of solutions):

$$x = 18^2 + 13 \times 5^2 = 324 + 325 = 649$$

$$y = 2 \times 18 \times 5$$

So the final solution is:

$$x = 649, y = 180$$

# Step-by-Step Solution Using Chakravala Method



## So the final solution is:

$$x = 649, y = 180$$

## Verification:

$$x^2 - 13y^2 = 649^2 - 13 \times 180^2 = 421201 - 421200 = 1$$