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Lecture No. 11: Module 1: Arithmetic, Algebra and Combinatorics

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Contents

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Indian Mathematics

- 1 The Zero and the Decimal System: The early appearance of Zero
- 2 Terms for the multiples of ten like 10, 20, 30 etc. in Rigveda. Terms for the higher powers of 10, given by Aryabhat, Mahaviracharya and Bhaskaracharya
- 3 The elementary operations like addition, subtraction, multiplication, division. Operations with fractions. Operations with zero. Squares and Cubes.
- 4 Methods to Obtain Square Roots and Cube Roots in Indian Mathematics
- 5 Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

Contents

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Dhamone

Subject Teacher
Santosh Dhamone

Indian Mathematics

- 6 Impossibility of square root of negative numbers, expressed by Indian mathematicians
- 7 Varga-Sankramana, Quadratic Equation
- 8 Trairashik, Vyasta-Trairashik, Paanchrashik, Saaptarashik
- 9 The problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.
- 10 The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Contents

My Inspiration
Late. Shival
Dhamone

Subject Teacher
Santosh Dhamone

Indian Mathematics

- 11 Step-by-Step Solution Using Chakravala Method
- 12 Progressions and Series in Indian Mathematics
- 13 Combinatorics in Ancient Indian Mathematics
- 14 Some examples from ancient Indian combinatorics with their original Sanskrit verses, followed by modern translations and explanations.

Step-by-Step Solution Using Chakravala Method

Find integer solutions to $x^2 - 13y^2 = 1$:

i.e., solve: $x^2 = 13y^2 + 1$:

Step 1: Choose initial triple (a, b, k)

We look for small integers a, b such that
 $a^2 - 13b^2 = k$

Try:

Let $a = 7, b = 2$

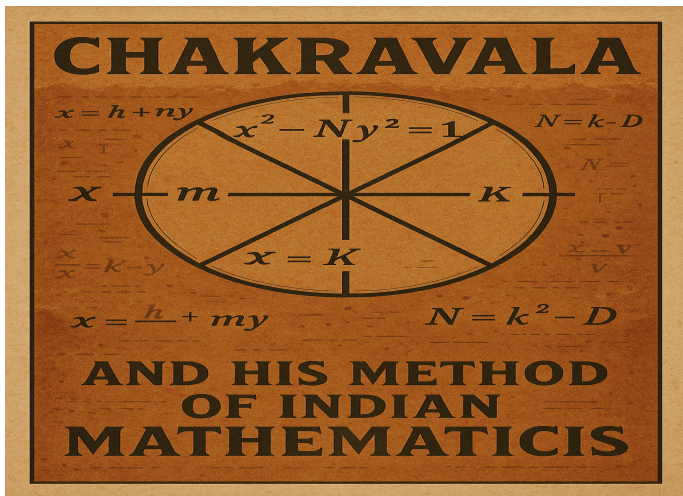
$$7^2 - 13 \times 2^2 = 49 - 52 = -3$$

$$\implies (a, b, k) = (7, 2, -3)$$

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Step 2: Find integer m such that

- $(a + bm)$ divisible by k , and
- $m^2 \equiv 13 \pmod{k}$
- Minimize $|m^2 - 13|$

Here $k = -3$, so:

Try $m = 1$;

$$7 + 21 = 28, 28(-3) = -84 \implies \text{divisible}$$

so, take $m=1$

Step-by-Step Solution Using Chakravala Method

Step 3: Compute next values

Use formulas:

$$a' = \frac{am + 13b}{|k|}, b' = \frac{a + bm}{|k|}, k' = \frac{m^2 - 13}{k}$$

Now plug in: $a = 7, b = 2, k = 3, m = 1$

$$a' = \frac{71 + 132}{|-3|} = \frac{7 + 26}{3} = \frac{33}{3} = 11$$

$$b' = \frac{7 + 21}{|-3|} = \frac{7 + 2}{3} = \frac{9}{3} = 3$$

$$k' = \frac{1^2 - 13}{-3} = \frac{1 - 13}{-3} = \frac{-12}{-3} = 4$$

Step-by-Step Solution Using Chakravala Method

New triple: $(a, b, k) = (11, 3, 4)$

Step 4: Repeat with new triple

Now: $(a = 11, b = 3, k = 4)$

Find m such that:

$$11 + 3m \equiv 0 \pmod{4} \rightarrow 3m \equiv -11 \equiv 1 \pmod{4}$$

$$\text{so } 3m \equiv 1 \pmod{4} \implies m \equiv 3 \pmod{4}$$

Try $m = 3$:

$$\text{Check } |m^2 - 13| = |9 - 13| = 4$$

Okay, take $m = 3$

Step-by-Step Solution Using Chakravala Method

New triple: $(a, b, k) = (11, 3, 4)$

Step 4: Repeat with new triple

Now: $(a = 11, b = 3, k = 4)$

Find m such that:

$$11 + 3m \equiv 0 \pmod{4} \rightarrow 3m \equiv -11 \equiv 1 \pmod{4}$$

$$\text{so } 3m \equiv 1 \pmod{4} \implies m \equiv 3 \pmod{4}$$

Try $m = 3$:

$$\text{Check } |m^2 - 13| = |9 - 13| = 4$$

Okay, take $m = 3$

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Step 5: Compute new triple

$$a' = \frac{11.3 + 133}{4} = \frac{33 + 39}{4} = \frac{72}{4} = 18$$

$$b' = \frac{11 + 33}{4} = \frac{11 + 9}{4} = \frac{20}{4} = 5$$

$$k' = \frac{3^2 - 13}{4} = \frac{9 - 13}{4} = \frac{-4}{4} = -1$$

New triple: $(a, b, k) = (18, 5, -1)$

Step-by-Step Solution Using Chakravala Method

Final Step: We now have

$$a^2 - 13b^2 = -1 \implies 18^2 - 13 \times 5^2 = 324 - 325 = -1$$

To get a solution for: $x^2 - 13y^2 = 1$

Multiply the current solution with itself (composition of solutions):

$$x = 18^2 + 13 \times 5^2 = 324 + 325 = 649$$

$$y = 2 \times 18 \times 5$$

So the final solution is:

$$x = 649, y = 180$$

Step-by-Step Solution Using Chakravala Method

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So the final solution is:

$$x = 649, y = 180$$

Verification:

$$x^2 - 13y^2 = 649^2 - 13 \times 180^2 = 421201 - 421200 = 1$$