Lecture No. 3: Module 1: Arithmetic, Algebra and Combinatorics

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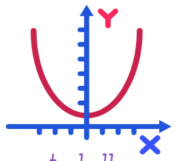

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1. The Zero and the Decimal System: The early appearance of Zero
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3. The elementary operations like addition, subtraction, multiplication, division. Operations with fractions. Operations with zero. Squares and Cubes.
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Elementary Operations and Their Development in Indian Mathematics:

Introduction:

Indian mathematics has a rich tradition of developing foundational arithmetic operations much before they were formalized in Europe. Early Indian mathematicians and texts, such as the **Sulba Sutras, Aryabhatiya, Ganita Sara Sangraha, and Lilavati**, show a sophisticated understanding of elementary operations, including those with fractions, zero, squares, and cubes. These operations form the bedrock of all mathematical computations.

Elementary Operations in Indian Mathematics:

a) Addition (Sankalana /):

- Addition was considered a fundamental operation and was well-defined by early Indian mathematicians.
- The term Sankalana or Yoga was used.
- In Lilavati, Bhaskaracharya provides both algorithms and poetic problems for adding numbers.
- Place-value notation and vertical alignment were used for large number addition.

Example:

"Add the numbers placed in rows as units, tens, hundreds, etc., carrying over as required" – Lilavati.

Elementary Operations in Indian Mathematics:

b) Subtraction (Vyavakalanam /):

- Known as Vyavakalanam or Apavartana.
- Used both in practical problems (like profit-loss, inheritance) and abstract algebraic reasoning.
- Negative results were known and accepted, showing maturity in concept development.

Notable:

Brahmagupta (7th century CE) clearly defined rules for subtraction including cases with negative results.

Elementary Operations in Indian Mathematics:

c) Multiplication (Guna /):

- Multiplication was viewed as repeated addition.
- Known as Guna or Gunakarana.
- Techniques included the Bhuta Sankhya system and positional multiplication.
- Special multiplication algorithms were used for large numbers, akin to lattice or column methods.

Example:

Aryabhata used tables and verse algorithms for efficient multiplication, even with large astronomical quantities.

Elementary Operations in Indian Mathematics:

d) Division (Bhaga /):

- Division (Bhaga, Harana) was treated as the inverse of multiplication.
- Methods for long division, remainder, and quotient were clearly described.
- Kuttaka method (pulverizer) was a special division method to solve indeterminate equations.

Brahmagupta's Rule for Division by Zero::

"A number divided by zero gives an infinite result" – though this concept was not in the modern calculus sense.

Operations with Fractions (Bhinnasankhya):

a) Recognition and Use:

- Fractions were called Bhinnasankhya.
- Indian texts described proper, improper, and mixed fractions.
- Operations like addition, subtraction, multiplication, and division were well defined.

b) Notation:

Numerators were written above and denominators below with a horizontal line, similar to modern notation.

Operations with Fractions (Bhinnasankhya):

c) Rules for Operations:

- **Addition/Subtraction:** Done using a common denominator.
- **Multiplication:** Numerators and denominators multiplied directly.
- **Division:** Inversion of divisor followed by multiplication (reciprocal method).

Mahaviracharya's text "Ganita Sara Sangraha" gives detailed procedures.

Operations with Zero (Shunya /):

a) Introduction of Zero:

- The concept of Shunya (Zero) was a groundbreaking contribution of Indian mathematics.
- First formalized by Brahmagupta in his work Brahmasphutasiddhanta (628 CE).

b) Rules for Operations with Zero:

Operation	**Result**
$a + 0$	a
$a - 0$	a
$a \times 0$	0
$a \div 0$	Undefined (Brahmagupta said ∞)
$0 \div a$	0

Squares and Cubes (Varga and Ghanam):

a) Square (Varga):

- Squaring was called Varga.
- Square numbers were used in geometry, particularly in Sulba Sutras for constructing altars.
- Aryabhata gave formulas for summation of square numbers.

Examples:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6}$$

Squares and Cubes (Varga and Ghanam):

b) Cube (Ghanam):

- Cubing was known as Ghana.
- Bhaskaracharya and Mahaviracharya gave procedures for cube root extraction and cube summation.

Examples:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\sum_{n=1}^{k} n^3 = \left(\frac{k(k+1)}{2}\right)^2$$

Squares and Cubes (Varga and Ghanam):

Summary Table of Terms:

Concept	**Sanskrit Term**	**Contributor(s)**
Addition	Sankalana / Yoga	Bhaskaracharya (Lilava
Subtraction	Vyavakalanam	Brahmagupta
Multiplication	Guna	Aryabhata, Mahavira
Division	Bhaga / Harana	Brahmagupta, Aryabha
Fractions	Bhinnasankhya	Mahaviracharya
Zero	Shunya	Brahmagupta
Square	Varga	Sulba Sutras, Aryabhat
Cube	Ghana	Bhaskaracharya, Maha