Lecture No. 4: Module 1: Arithmetic, Algebra and Combinatorics

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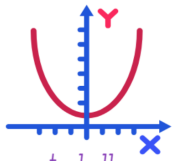

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1. The Zero and the Decimal System: The early appearance of Zero
2. Terms for the multiples of ten like 10, 20, 30 etc. in Rigveda. Terms for the higher powers of 10, given by Aryabhat, Mahaviracharya and Bhaskaracharya
3. The elementary operations like addition, subtraction, multiplication, division. Operations with fractions. Operations with zero. Squares and Cubes.
4. Methods to Obtain Square Roots and Cube Roots in Indian Mathematics
5. Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

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Methods to Obtain Square Roots and Cube Roots in Indian Mathematics:

Introduction:

Ancient Indian mathematicians developed highly accurate and systematic methods to extract square roots and cube roots, long before such methods were formalized in Europe. Two prominent mathematicians in this tradition were:

- Aryabhata (476 CE) – through his work Aryabhatiya
- Bhaskaracharya (1114–1185 CE) – through his works Lilavati and Bijaganita

Their methods were algorithmic in nature and involved step-by-step computations similar to modern root

Indian Mathematics

Methods to Obtain Square Roots and Cube Roots in Indian Mathematics:Square Root Extraction (Varga Moola):

1) Aryabhata's Method for Square Root:

Aryabhata describes a method to find the square root in Aryabhatiya (Stanza 5 of Ganitapada). It is an approximation method based on place-value, somewhat similar to the modern long division method.

Method Outline:

Let the number be . The steps include:

Indian Mathematics

Methods to Obtain Square Roots and Cube Roots in Indian Mathematics:Square Root Extraction (Varga Moola):

1. Group the digits in pairs from right to left.
2. Find the largest square $\leq$ leftmost group — this gives the first digit of the root.
3. Subtract square of the root from that group and bring down the next pair.
4. Double the current root, and find the next digit that fits.
5. Repeat until all digits are used.

Indian Mathematics

Methods to Obtain Square Roots and Cube Roots in Indian Mathematics:Square Root Extraction (Varga Moola):

Example:

To find $\sqrt{1522756}$

1. Group: $15|22|75|6$
2. Start with 3 (since $3^2 = 9 < 15$); *remainder* $= 6$; bring down $22 \rightarrow 622$
3. Double $3 = 6$; next digit $= 8 \rightarrow 68 \times 8 = 544 < 622$
4. Continue the process ...

Aryabhata's method gives digits one by one, similar to modern manual square root extraction.

Indian Mathematics

Methods to Obtain Square Roots and Cube Roots in Indian Mathematics:Square Root Extraction (Varga Moola):

2) Bhaskaracharya's Method for Square Root:

In Lilavati, Bhaskara presents a more refined step-by-step rule in verse, suitable for teaching.

Key Concepts:

- Break the number into even-digit groups from units.
- Use trial and error, subtraction, and remainder to find each digit.
- Involve squaring and estimating the next digit based on the current approximation.

Indian Mathematics

Methods to Obtain Square Roots and Cube Roots in Indian Mathematics:Square Root Extraction (Varga Moola):

2) Bhaskaracharya's Method for Square Root:

Verse (Lilavati):

"Subtract the greatest square from the first group, take the square root. Double it, write below. Divide the remaining group with this, get the next digit . . ."

Strengths of Bhaskara's Method:

- Detailed and educational
- Accurate for both whole numbers and decimals
- Adaptable for irrational square roots with

Indian Mathematics

Cube Root Extraction (Ghana Moola):

1) Aryabhata's Cube Root Method:

Though Aryabhata primarily focused on astronomy, cube root extraction was known in his period and inferred from his work. The method follows logic similar to square roots but adapted for cubes.

General Steps::

1. Group digits in threes from the right.
2. Find the largest cube leftmost group.
3. Subtract, bring down the next group.
4. Use an estimation formula to find the next digit: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
5. Extract digits one at a time.

Indian Mathematics

Cube Root Extraction (Ghana Moola):

2) Bhaskaracharya's Cube Root Method:

In *Lilavati*, Bhaskaracharya gives a precise method for extracting cube roots — explained in poetic Sanskrit, but algorithmically sound.

Step-by-Step Process::

1. Group the number into triads (groups of 3 digits) starting from the unit place.
2. Estimate the first digit x such that x^3 is just less than or equal to the first group.
3. Subtract x^3 and bring down the next group.
4. Use: $3x^2\dot{y} + 3x\dot{y}^2 + y^3$ to estimate the next digit.
5. Continue the process to extract each digit of the