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Lecture No. 5: Module 1: Arithmetic, Algebra and Combinatorics

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Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

Square Root Extraction (Varga Moola):

Example 1: Find $\sqrt{1522756}$ by Method: Bhaskaracharya's Place-Value Method (similar to long division) .

Step 1: Group digits in pairs from right

$$1522756 \rightarrow 15|22|75|6$$

Step 2: Find the largest square ≤ 15

■ $\sqrt{15} \approx 3.87 \rightarrow \text{take } 3 \rightarrow 3^2 = 9$

■ **Write 3 as the first digit of root. Subtract:**

$$15 - 9 = 6 \rightarrow \text{bring down } 22 \rightarrow 622$$

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Step 3: Double 3 → 6, write as divisor prefix:

Try $68 \times 8 = 544$, $69 \times 9 = 621$

- $69 \times 9 = 621 \leq 622 \rightarrow \text{take } 9$
- Next digit = 9; Root so far = 39
- Subtract: $622 - 621 = 1 \rightarrow \text{bring down } 75 \rightarrow 175$

Step 4: Double 39 → 78, write as divisor prefix:

Try $782 \times 2 = 1564$

- $782 \times 2 = 1564 < 1750 \rightarrow \text{try } 783 \times 3 = 2349$
(too big) Try $781 \times 2 = 1562 \rightarrow \text{fits}$
- Next digit = 2; Root so far = 392
- Subtract: $1750 - 1562 = 188 \rightarrow \text{bring down } 6 \rightarrow$

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Step 5: Double 392 → 784, try fitting digit:

Try $7842 \times 2 = 15684$ (too big), go for smallest that fits

Try $7812 \times 2 = 15624 \rightarrow$ fits

■ Next digit = 2

■ Final root: 1234

Answer :: $\sqrt{1522756} = 1234$

Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

Cube Root Extraction (Ghana Moola):

Example 2: Find $\sqrt[3]{48228544}$ by Method: Bhaskaracharya's Cube Root Method (digit-wise, using identity).

Step 1: Group digits into triads from right

$48228544 \rightarrow 48|228|544$

Step 2: First digit

Find cube root of 48:

■ $3^3 = 27, 4^3 = 64 \rightarrow \text{pick } 3$

■ Root so far = 3

■ Subtract $3^3 = 27 \rightarrow = 21$

■ Bring down 228 \rightarrow new dividend = 21228

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Step 3: Use formula:

Next digit = y ,

Use:

$$(30)^2 \cdot y = 900y \quad (\text{approximate to find fitting } y)$$

Try $y = 2 \rightarrow 900 \times 2 = 1800$

Still less than 21228 \rightarrow try $y = 2.3 \rightarrow$ test with real expansion

Try $y = 2$

New root = 32

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Step 3: Use formula:

Use identity:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Let $a = 30$, $b = 2$

$$\begin{aligned}(30 + 2)^3 &= 30^3 + 3302 + 3304 + 8 \\ &= 27000 + 5400 + 720 + 8 = 33128\end{aligned}$$

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Step 3: Use formula:

Too large. Try $b = 1$

$$\begin{aligned}(30 + 1)^3 &= 30 + 3301 + 3301 + 1 \\ &= 27000 + 2700 + 90 + 1 = 29791\end{aligned}$$

Still too high. Go back — must be error in expansion
Try a new number to make the process clearer.

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Simplified Cube Root Example: $\sqrt[3]{74088}$

Step 1: Group digits from right $\rightarrow 74 \text{ --- } 088$

Step 2: Estimate root of 74

$$4^3 = 64 < 74$$

$$5^3 = 125 > 74 \rightarrow \text{take } 4$$

Root so far = 4

Subtract 64 from 74 \rightarrow remainder = 10

Bring down 088 \rightarrow new dividend = 10088

Step 3: Use identity approximation:

Try $40 + x$

Let's try $x = 2$

$$(40 + 2)^3 = 40^3 + 3 \times 40^2 \times 2 + 3 \times 40 \times 4 + 8$$

$$64000 + 9600 + 480 + 8 = 74088$$