Lecture No. 5: Module 1: Arithmetic, Algebra and Combinatorics

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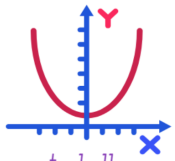

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Indian Mathematics

Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

Square Root Extraction (Varga Moola):

Example 1: Find $\sqrt{1522756}$ by Method: Bhaskaracharya's Place-Value Method (similar to long division) .

Step 1: Group digits in pairs from right

$1522756 \rightarrow 15|22|75|6$

Step 2: Find the largest square ≤ 15

- $\sqrt{15} \approx 3.87 \rightarrow take3 \rightarrow 3^2 = 9$
- **Write 3 as the first digit of root. Subtract:** $15 - 9 = 6 \rightarrow$ **bring down** $22 \rightarrow 622$

Square and Cube Roots by Aryabhata Bhaskaracharya

Step 3: Double 3 → 6, write as divisor prefix:
Try $68 \times 8 = 544, 69 \times 9 = 621$

- $69 \times 9 = 621 \leq 622 \rightarrow$ *take* 9
- Next digit = 9; Root so far = 39
- Subtract: 622 − 621 = 1 → bring down 75 → 175

Step 4: Double 39 → 78, write as divisor prefix:
Try $782 \times 2 = 1564$

- $782 \times 2 = 1564 < 1750 \rightarrow$ *try* $783 \times 3 = 2349$ (too big) Try $781 \times 2 = 1562 \rightarrow$ *fits*
- Next digit = 2; Root so far = 392
- Subtract: 1750 − 1562 = 188 → bring down 6 → 1886

Square and Cube Roots by Aryabhata Bhaskaracharya

Step 5: Double 392 $\rightarrow$ 784, try fitting digit:

Try $7842 \times 2 = 15684$ (too big), go for smallest that fits

Try $7812 \times 2 = 15624 \rightarrow$ fits

- Next digit $= 2$
- Final root: 1234

Answer :: $\sqrt{1522756} = 1234$

Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

Cube Root Extraction (Ghana Moola):

Example 2: Find $\sqrt[3]{48228544}$ by Method: Bhaskaracharya's Cube Root Method (digit-wise, using identity).

Step 1: Group digits into triads from right

$48228544 \rightarrow 48|228|544$

Step 2: First digit

Find cube root of 48:

- $3^3 = 27, 4^3 = 64 \rightarrow$ *pick* 3
- Root so far $= 3$
- Subtract $3^3 = 27 \rightarrow = 21$
- Bring down 228 $\rightarrow$ *new dividend* $= 21228$

Square and Cube Roots by Aryabhata Bhaskaracharya

Step 3: Use formula:

Next digit $= y$,

Use:

$$(30)^2 \cdot y = 900y \quad \text{(approximate to find fitting y)}$$

Try $y = 2 \rightarrow 900 \times 2 = 1800$

Still less than $21228 \rightarrow$ try $y = 2.3 \rightarrow$ test with real expansion

Try $y = 2$

New root $= 32$

Square and Cube Roots by Aryabhata Bhaskaracharya

Step 3: Use formula:

Use identity:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Let a = 30, b = 2

$$(30+2)^3 = 30^3 + 3302 + 3304 + 8$$

$$= 27000 + 5400 + 720 + 8 = 33128$$

Square and Cube Roots by Aryabhata Bhaskaracharya

Step 3: Use formula:

Too large. Try b = 1

$$(30+1)^3 = 30 + 3301 + 3301 + 1$$

$$= 27000 + 2700 + 90 + 1 = 29791$$

Still too high. Go back — must be error in expansion
Try a new number to make the process clearer.

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Simplified Cube Root Example: $\sqrt[3]{74088}$

Step 1: Group digits from right $\rightarrow$ 74 — 088

Step 2: Estimate root of 74

$4^3 = 64 < 74$

$5^3 = 125 > 74 \rightarrow$ take 4

Root so far $= 4$

Subtract 64 from 74 $\rightarrow$ remainder $= 10$

Bring down 088 $\rightarrow$ new dividend $= 10088$

Step 3: Use identity approximation:

Try $40 + x$

Let's try $x = 2$

$(40 + 2)^3 = 40^3 + 3\times40^2\times2 + 3\times40\times4 + 8$

$64000 + 9600 + 480 + 8 = 74088$