My Inspiration
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Lecture No. 6: Module 1: Arithmetic, Algebra and Combinatorics

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Contents

Indian Mathematics

Contents

Indian Mathematics

Contents

Indian Mathematics

Introduction:

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In the realm of classical Indian mathematics, great scholars such as Brahmagupta, Bhaskaracharya, and others contributed significantly to the development of algebra, arithmetic, and number theory. Their discussions included both positive and negative numbers, and the rules of operations on them.

However, they did not recognize the square root of a negative number as a valid or meaningful number. This idea — now fundamental in modern mathematics through the concept of imaginary numbers (e.g., $i = \sqrt{-1}$) was absent in Indian mathematical literature.

Understanding the Concept of Square Roots in Ancient India:

In classical Indian mathematics:

- The square root of a number x, denoted $\sqrt{x}$, is defined only if $x \geq 0$
- The result of taking a square root was expected to be real and positive (or occasionally negative when symmetry was needed).
- Mathematicians worked primarily within the scope of positive rational and irrational numbers.

Brahmagupta (7^{th} Century CE):

In classical Indian mathematics:

Brahmagupta, in his seminal work Brahmasphutasiddhanta, introduced zero and negative numbers into mathematics. He formulated rules for operations with positive and negative numbers (referred to as "fortune" and "debt", respectively).

Key Contributions of Brahmagupta:

- Rules for addition, subtraction, multiplication, and division involving zero and negatives.
- Recognized that a negative number multiplied by

Key Contributions of Brahmagupt

- Recognized that a negative number multiplied by another negative gives a positive result.

Square Roots of Negative Numbers:

Brahmagupta explicitly stated that a negative number does not have a square root.
Quote (paraphrased from his text): *"A negative quantity cannot have a square root, as there is no number which when squared gives a negative result."*
This shows that Brahmagupta did not admit solutions to $\sqrt{-x}$ for $x > 0$. In his framework, such an operation was undefined or impossible.

Bhaskaracharya (12^{th} Century CE):

In classical Indian mathematics:

Bhaskaracharya (Bhaskara II), in his famous work Lilavati, extended Brahmagupta's rules and gave examples of solving quadratic equations. He understood the concept of discriminant $b^2 - 4ac$ in quadratic equations.

Important Point::

- If the discriminant $b^2 - 4ac$, Bhaskara dismissed the solution, stating that no real root exists.
- He called such results "asambhavit" (), meaning "impossible" or "inconceivable."

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Bhaskaracharya (12^{th} Century CE):

Example:

Suppose a quadratic equation gives the solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac < 0$ Bhaskara would conclude:
"There is no valid solution in this case."
So, while Bhaskara worked with irrational square roots (like $\sqrt{2}, \sqrt{3}$), he rejected square roots of negative values.

Philosophical and Mathematical Reasoning:

In Indian philosophy and mathematics:

- Numbers were tied to quantities and measurements, especially in astronomy, geometry, and commerce.
- A square root was viewed as the side of a square with a known area.
- Since no square has a negative area, the square root of a negative number was seen as meaningless.

Thus, the absence of $\sqrt{-1}$ in Indian mathematics is not a weakness, but a reflection of the real-number-centric worldview of ancient scholars.

Historical Perspective: Imaginary Numbers Came Later:

In Indian philosophy and mathematics:

- The concept of imaginary numbers was developed in 16th-century Europe, over 800 years after Bhaskara.
- Mathematicians like Cardano, Euler, and Gauss introduced and formalized the idea of $i = \sqrt{-1}$.
- This concept was completely absent in classical Indian, Greek, or Arabic mathematical texts.