My Inspiration
Late. Shivlal
Dhamone

Subject Teacher
Santosh Dhamone

Lecture No. 7: Module 1: Arithmetic, Algebra and Combinatorics

Subject Teacher
Santosh Dhamone

Assistant Professor in Mathematics
Art's Commerce and Science College,Onde
Tal:- Vikramgad, Dist:- Palghar

ssdhamone@acscollegeonde.ac.in
www.santoshdhamone.com

July 5, 2025

Contents

Indian Mathematics

1. The Zero and the Decimal System: The early appearance of Zero
2. Terms for the multiples of ten like 10, 20, 30 etc. in Rigveda. Terms for the higher powers of 10, given by Aryabhat, Mahaviracharya and Bhaskaracharya
3. The elementary operations like addition, subtraction, multiplication, division. Operations with fractions. Operations with zero. Squares and Cubes.
4. Methods to Obtain Square Roots and Cube Roots in Indian Mathematics
5. Solved Examples: Square and Cube Roots by Aryabhata Bhaskaracharya

Contents

Indian Mathematics

6. Impossibility of square root of negative numbers, expressed by Indian mathematicians

7. Varga-Sankramana, Quadratic Equation

8. Trairashik, Vyasta-Trairashik, Paanchrashik, Saaptarashik

9. The problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

10. The Problem of Varga Prakriti and the Method Given by Bhaskaracharya

Contents

Indian Mathematics

- 11 Step-by-Step Solution Using Chakravala Method
- 12 Progressions and Series in Indian Mathematics
- 13 Combinatorics in Ancient Indian Mathematics
- 14 Some examples from ancient Indian combinatorics with their original Sanskrit verses, followed by modern translations and explanations.

Varga-Sankramana and Quadratic Equations in Indian Mathematics:
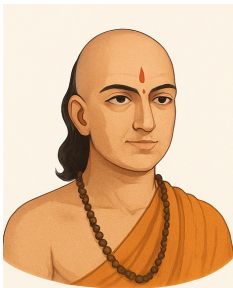
Introduction:

Quadratic equations have a long and rich history in Indian mathematics, dating back to the early classical period. The ancient Indian method of solving quadratic equations is known as "Varga-Sankramana" ( ), which roughly translates to "transposition of squares" or "shifting the square term".

Ancient Indian mathematicians such as Brahmagupta and Bhaskaracharya developed systematic techniques for solving quadratic equations using verbal and algebraic rules, centuries before they were formalized in Europe.

ARYABHATA
(c. 476–550)

Aryabhata was an ancien lin-
an mathematician and astronomer.
In his work 'Aryabhatiya,' he wrote
about mathematics, astronomy, and
the concept of zero. He introduced
the place value system in India and
made pioneering calculations of
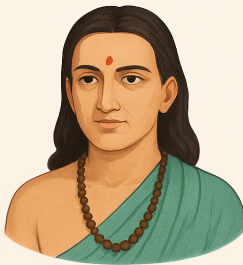
MAHAVIRA
(c. 815–885)

Mahavira was an Indian mathe-
matician who made significant
contributions to algebra and
geometry. In his major work, '*Gan-
itasarasamgraha*,' he presented
systematic rules for solving quad-
ratic, cubic, and quartic equations.
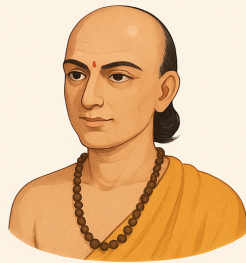
BHASKARACHARYA
(c. 1114–1185)

Bhaskaracharya was a talented
Indian mathematician and astrono-
mer. His renowned works. '*Siddhanta
Shiromani*' and '*Lilavati*,' cover topics
such as algebra, arithmetic, geomet-
try and planetary cnathematics. He
also made significant contributions

Varga-Sankramana and Quadratic Equations in Indian Mathematics:

Terminology:

Sanskrit Term	Meaning	Modern Equiv
Varga	Square	x^2
Mula	Root (or unknown)	x
Rāshi	Quantity/Number	Constant term
Sankramana	Transformation transposition	Moving terms across ” = ”
Varga-Sankramana	Square transposition	Rearranging a quadratic eqn

General Form of Quadratic Equation:

Modern quadratic equation:
$ax^2 + bx = c$ or $ax^2 + bx + c = 0$

Indian mathematicians expressed it in rhetorical (verbal) form. For example:

"The square of a number, added to 5 times the number, is equal to 6."

This corresponds to:

$$x^2 + 5x = 6 \implies x^2 + 5x + 6 = 0$$

This transformation or rearrangement process is called **Varga-Sankramana**.

Varga-Sankramana and Quadratic Equations in Indian Mathematics

My Inspiration
Late. Shivlal
Dhamone

Subject Teacher
Santosh Dhamone

Brahmagupta's Contribution (7th century CE):

In Brahmasphutasiddhanta, Brahmagupta gives a rule to solve the quadratic equation of the form:
$$ax^2 + bx = c \text{ or } ax^2 + bx + c = 0$$

Rule (in words):

"To the square root of the square of half the coefficient of the unknown, add or subtract the root of the constant term; then divide by the coefficient of the unknown."
This is equivalent to the modern formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

However, Brahmagupta only considered real roots. If the discriminant was negative $b^2 - 4ac < 0$, he called it **asambhavit** (impossible).

Varga-Sankramana and Quadratic Equations in Indian Mathematics

Bhaskaracharya's Contribution ($12^t h$ century CE):

Bhaskaracharya, in **Bijaganita**, further refined quadratic methods. He gave:

* Solutions to quadratic equations both with and without constant terms.
* Practical geometrical and word problems using quadratics.
* Rules for both positive and negative coefficients.

Example Problem (from Lilavati):

"The square of a number multiplied by 3, added to 5 times the number, gives 8."

$3x^2 + 5x = 8$ or $3x^2 + 5x - 8 = 0$

My Inspiration
Late. Shivlal
Dhamone

Subject Teacher
Santosh Dhamone

Varga-Sankramana and Quadratic Equations in Indian Mathematics

Indian Method of Completing the Square (Varga-Kriya):

Another technique known to Indian mathematicians was completing the square, used for solving quadratics:

Example: Solve $x^2 + 6x = 16$

Add $(\frac{6}{2})^2 = 9$ to both sides
$x^2 + 6x + 9 = 16 + 9$
$\implies (x+3)^2 = 25$
$\implies (x+3) = \pm 5$
$\implies x + 3 = 5$ or $x + 3 = -5$
$\implies x = 2$ or $x = -8$

This completing the square method was known to both Brahmagupta and Bhaskara, even without symbolic notation

Example 1: Solve: "The square of a number plus 5 times the number is equal to 6"

Let the number be x:

$x^2 + 5x = 6 \implies x^2 + 5x - 6 = 0$

Equating with $ax^2 + bx + c = 0$ we get;

$a = 1, b = 5, c = -6$

Use the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\implies x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times -6}}{2 \times 1}$

$\implies x = \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2} = \frac{-5 \pm 7}{2}$

$\implies x = \frac{-5 + 7}{2}$ or $x = \frac{-5 - 7}{2}$

$\implies x = \frac{2}{2}$ or $x = \frac{-12}{2}$

$\implies x = 1$ or $x = -6$ Required Solution.

Example 2: Solve: "3 times the square of a number plus 5 times the number is equal to 8"

Let the number be x:

$3x^2 + 5x = 8 \implies 3x^2 + 5x - 8 = 0$

Equating with $ax^2 + bx + c = 0$ we get;

$a = 3, b = 5, c = -8$

Use the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\implies x = \frac{-5 \pm \sqrt{5^2 - 4 \times 3 \times -8}}{2 \times 3}$

$\implies x = \frac{-5 \pm \sqrt{25 + 96}}{6} = \frac{-5 \pm \sqrt{121}}{6} = \frac{-5 \pm 11}{6}$

$\implies x = \frac{-5 + 11}{6}$ or $x = \frac{-5 - 11}{6}$

$\implies x = \frac{6}{6}$ or $x = \frac{-16}{6}$

$\implies x = 1$ or $x = \frac{-8}{3}$ Required Solution.

Varga-Sankramana and Quadratic Equations in Indian Mathematics

Legacy and Influence:

* These early works on Varga-Sankramana anticipated later developments in algebraic theory.

* Indian methods were rule-based, algorithmic, and geometrically visualized.

* They influenced later Islamic and European mathematicians through translation and transmission.

Conclusion:

The Varga-Sankramana method represents an early and powerful algebraic technique in Indian mathematics, used to solve quadratic equations systematically. The works of Brahmagupta and Bhaskaracharya remain milestones in the history of algebra, showcasing a deep understanding of mathematical logic, even without symbolic notation.