Lecture No. 9: Module 1: Arithmetic, Algebra and Combinatorics

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Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Introduction to Kuttaka () Problem:

Introduction:

The **Kuttaka** (meaning "pulverizer" in Sanskrit) problem refers to a class of problems in number theory that involve finding integer solutions to **linear indeterminate equations**, especially **Diophantine equations** of the form: $ax + c = by$
Where:

- a, b, c are are known integers,
- x, y are unknown integers to be determined.

The problem reduces to solving the equation for integers x and y.

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Origin and Significance:

- Originated in ancient Indian mathematics, particularly in the works of Aryabhata I (476 CE), and later refined by Brahmagupta (7th century) and Bhaskaracharya II (12th century).
- The method was essential for calendar calculations, astronomy, and modular arithmetic.

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Brahmagupta's Contribution:

A. General Linear Indeterminate Equation

Brahmagupta dealt with equations of the form:
$ax + c = by$ or rearranged as: $ax - by = -c$

B. Brahmagupta's Method (Kuttaka Algorithm)

Brahmagupta provided a systematic way to solve this equation using the Kuttaka method, which involved:

- Euclidean algorithm to find the greatest common divisor (GCD) of a and b,
- Back-substitution to find integral solutions.

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Brahmagupta's Contribution:

Steps in the Method:

- Reduce the equation to $ax + c = by$ form.
- **Check divisibility**: Ensure $\gcd(a, b)$ divides c, otherwise no integer solution exists.
- **Apply the Euclidean algorithm** to compute the GCD of a and b.
- **Back-substitute** through the steps of Euclidean algorithm to express GCD as a linear combination of a and b.
- Use this relation to find one solution (x_0, y_0), and then the general solution using integer parameters.

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Bhaskaracharya's Contribution:

A. Refinement of Brahmagupta's Kuttaka:

- In **Bijaganita** and **Lilavati**, Bhaskaracharya provided clearer algorithms and worked examples.
- Introduced improvements in clarity and organization of the method.
- Emphasized practical computation steps, using tables and modular arithmetic.

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Bhaskaracharya's Contribution:

B. Bhaskara's Simplified Rules:

Bhaskara:

- Extended the solution technique to congruence equations and modular arithmetic.
- Introduced the "cyclic method" (Chakravala) for more complex indeterminate equations (especially quadratic Pell-type).

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Worked Example:

Problem: Solve $26x + 3 = 65y$:

Bhaskara:

- Step 1: Rearranged as:
  $26x - 65y = -3$
- Step 2: Find $\text{GCD}(26, 65)$
  Using Euclidean algorithm:
  - $65 \div 26 = 2$, remainder 13
  - $26 \div 13 = 2$, remainder 0

  $\rightarrow$ gcd=13
  Check if 13 divides -3: No.
  Hence, no integer solution exists.

Problem of Kuttaka and the methods given by Brahmagupta and Bhaskaracharya.

Worked Example:

Problem: Solve $26x + 13 = 65y$:

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  $26x - 65y = -13$
- Step 2: Find GCD$(26, 65)$
  Using Euclidean algorithm:
  * $65 \div 26 = 2$, remainder 13
  * $26 \div 13 = 2$, remainder 0

  $\rightarrow$ gcd=13
  Now proceed with back substitution:

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solution continue:

From Euclidean steps:

- $65 = 2 \times 26 + 13$
- $26 = 2 \times 13 + 0$

$13 = 652 \times 2613 = 65 + 2 \times 26$
Multiply both sides by 1:
$13 = (1) \times 65 + 2 \times 26$
So,
$x_0 = 2, y_0 = 1$
General Solution:
$x = x_0 + (b/gcd)k = 2 + (65/13)k = 2 + 5k$
$y = y_0 + (a/gcd)k = 1 + (26/13)k = 1 + 2k$
where $k \in \mathbb{Z}$